FINAL

Davis
M211

Name:
Pledge:

Show all work; unjustified answers may receive less than full credit.

1. Suppose that the median price of a home in the U.S. was \$50,000 in 1970 and \$100,000 in 1990.

a. If prices are increasing linearly, find an equation for the line that represents price P in terms of time t (it is easier if you let t represent years since 1970). What will the median price be in the year 2000 under this assumption?

$P = 2.5t + 50$. The median price in 2000 would be \$125,000.

b. If prices are increasing exponentially, find an equation in the form $P = P_0 a^t$ which describes the price P as a function of t, the number of years after 1970. What will the median price be in the year 2000 under this assumption?

$P = (50)2^{t/20}$. The median price in 2000 would be \$140,000.

Which of these do you think is more realistic and why?

The exponential model is more realistic because of compounding affects in the housing market.

2. Use the definition of the derivative to show that $(\frac{1}{x})' = \frac{-1}{x^2}$. No credit for the short cut calculation.

$$(\frac{1}{x})' = lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h} = lim_{h\to 0}\frac{\frac{x-(x+h)}{x(x+h)}}{h} = lim_{h\to 0}\frac{-1}{x(x+h)} = \frac{-1}{x^2}.$$

3. Use any method you like (namely the short cut rules!) to find $\frac{dy}{dx}$ of the following functions.

a. $y = 3x^2 + 2x^3 - \frac{3}{2}x^{\frac{2}{3}}$ $\frac{dy}{dx} = 6x + 6x^2 - x^{-\frac{1}{3}}$

b. $y = e^x \sin(x)$

$\frac{dy}{dx} = e^x \sin(x) + e^x \cos(x)$

c. $y = \sin(e^x)$

$\frac{dy}{dx} = \cos(e^x)e^x$

d. $y = \frac{x^2+x+1}{\tan(x)}$

$$\frac{dy}{dx} = \frac{\tan(x)(2x+1)-(x^2+x+1)\frac{1}{\cos^2(x)}}{\tan^2(x)}$$

e. $y = Arctan(x^2)$

$\frac{dy}{dx} = \frac{2x}{1+x^4}$

4. Do one of the following two:

a. Use the definition of the derivative to show that $(x^n)' = nx^{n-1}$ for n a positive integer.

$$(x^n)' = lim_{h\to 0}\frac{(x+h)^n - x^n}{h} = lim_{h\to 0}\frac{x^n + nx^{n-1}h + junk(h^2) - x^n}{h} = lim_{h\to 0} nx^{n-1} + junk(h) = nx^{n-1}$$

b. Assuming part a. of this problem to be true, use the chain rule to show that $\frac{dy}{dx} = \frac{m}{n}x^{\frac{m}{n}-1}$ when $y = x^{\frac{m}{n}}$ for m and n positive integers.

$$y^n = x^m; ny^{n-1}\frac{dy}{dx} = mx^{m-1}; \frac{dy}{dx} = \frac{m}{n}\frac{x^{m-1}}{y^{n-1}} = \frac{m}{n}x^{m-1-\frac{m}{n}(n-1)} = \frac{m}{n}x^{\frac{m}{n}-1}.$$

5. Suppose that the population of the United States can be modeled by the function $f(t) = -.001t^3 + .15t^2 + .1t + 200$, where t is measured in years after 1970 and f is in millions of people. When is the population growing most rapidly (hint: make sure you first identify what you are trying to maximize!)? Calculate $f'(26)$ and $f''(26)$, and interpret what they mean.

Maximize the first derivative, so set the second derivative equal to 0: $f''(t) = -.006t + .3 = 0, t = 50$. Thus, the population will be growing the most rapidly in the year 2020. $f'(26) = 5.872$, which means that in the year 1996 roughly 5.8 million new people will be added to the population of the US. $f''(26) = .144$, which means the rate at which people are being added to the US will increase by about 140,000 people, so in 1997 the population growth will be approximately 6.016 million people per year.

6. A sketch of $y = f'(x)$ is on the board. Sketch the graph of $y = f(x)$ and $y = f''(x)$ on two different graphs. Draw three different possible graphs for the antiderivative.

7. A boat is traveling $v(t) = 64 - t^2$, where v is measured in feet per second.

a. Write a definite integral that expresses the total distance traveled in the first 8 seconds.

$\int_0^8 (64 - t^2)dt$

b. Compute a left hand sum with 4 rectangles to approximate the total distance traveled in 8 seconds. Is the left hand sum an overestimate or an underestimate? (Explain!!)

$64(2) + 60(2) + 48(2) + 28(2) = 400$ feet is an overestimate.

c. Find an antiderivative for $v(t)$, and use that to find the exact distance traveled in 8 seconds.

$64t - t^3/3|_0^8 = 512 - 512/3 = 341.33$ feet

8. You have a profit equation of $\pi(q) = -q^3 + 150q^2 - 5625q + 1000$, where q and $\pi(q)$ are measured in thousands. What value of q will maximize the profit? For what values of q will you make a profit (as opposed to losing money)? What value of q will maximize the marginal profit?

Setting the first derivative equal to 0, we get critical points of $q = 25$ and $q = 75$. It turns out that $q = 75$ will give a maximum profit of \$1000 (Side note: $q = 0$ will also produce a profit of \$1000, but this is not realistic and should be discarded)