Quiz 1

Davis
M212

Name:
Pledge:

(9pts.) 1. Use integration by parts to justify the following formula: $\int \sin^n(x)dx = -\frac{1}{n}\sin^{n-1}(x)\cos(x) + \frac{n-1}{n}\int \sin^{n-2}(x)dx$ for n positive. Use the formula to calculate $\int \sin^7(x)dx$.

$u = \sin^{n-1}(x); dv = \sin(x)$
$du = (n-1)\sin^{n-2}(x)\cos(x)dx; v = -\cos(x)$

$$\begin{aligned}\int \sin^n(x)dx &= -\sin^{n-1}(x)\cos(x) + \int (n-1)\sin^{n-2}(x)\cos(x)(\cos(x))dx \\ &= -\sin^{n-1}(x)\cos(x) + \int (n-1)\sin^{n-2}(x)\cos^2(x)dx \\ &= -\sin^{n-1}(x)\cos(x) + \int (n-1)\sin^{n-2}(x)(1-\sin^2(x))dx \\ &= -\sin^{n-1}(x)\cos(x) + (n-1)\int \sin^{n-2}(x)dx - (n-1)\int \sin^n(x)dx\end{aligned}$$

Adding $(n-1)\int \sin^n(x)dx$ to the both sides, we get $n\int \sin^n(x) = -\sin^{n-1}(x)\cos(x) + (n-1)\int \sin^{n-2}(x)dx$, so $\int \sin^n(x) = -\frac{1}{n}\sin^{n-1}(x)\cos(x) + \frac{n-1}{n}\int \sin^{n-2}(x)dx$ as claimed.

Applying this to the case where $n = 7$, we get $\int \sin^7(x)dx =$
$-\frac{1}{7}\sin^6(x)\cos(x) + \frac{6}{7}(-\frac{1}{5}\sin^4(x)\cos(x) + \frac{4}{5}(-\frac{1}{3}\sin^2(x)\cos(x) + \frac{2}{3}(-\cos(x))))$.

(6pts.) 2. Integrate the following:

a. $\int \sin(e^{2t})e^{2t}dt$

Setting $u = e^{2t}$, we get $du = 2e^{2t}dt$, so $\int \sin(e^{2t})e^{2t}dt = \frac{1}{2}\int \sin(u)du = -\frac{1}{2}\cos(u) + C = -\frac{1}{2}\cos(e^{2t}) + C$.

b. $\int \frac{t+2}{t^2-5t-6}dt$

Partial fractions: $\frac{t+2}{t^2-5t-6} = \frac{A}{t-6} + \frac{B}{t+1}$. Getting a common denominator of t^2-5t-6, we see that the numerators satisfy $t+2 = (A+B)t + (B-6A)$. Solving for A and B, we get $A = -\frac{1}{7}$ and $B = \frac{8}{7}$. Thus, $\int \frac{t+2}{t^2-5t-6}dt = \int(\frac{-\frac{1}{7}}{t-6} + \frac{\frac{8}{7}}{t+1})dt = -\frac{1}{7}\ln|t-6| + \frac{8}{7}\ln|t+1| + C$.

(5pts.) 3. Verify the following antiderivative by differentiation.

$\int \arctan x dx = x\arctan x - \frac{1}{2}\ln(1+x^2) + C$

$(x\arctan x - \frac{1}{2}\ln(1+x^2) + C)' = x\frac{1}{1+x^2} + \arctan x(1) - \frac{1}{2}\frac{2x}{1+x^2} = \arctan x$.