Quiz 2

Davis
M212

Name:
Pledge:

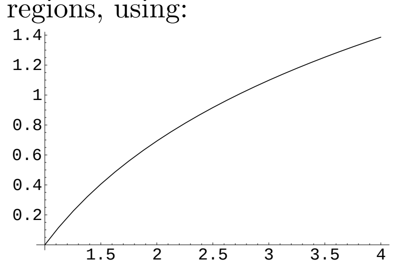
(8pts.) 1. Given the picture of $y = f(x)$, estimate the area under the curve from 1 to 4, subdividing the interval into 3 regions, using:

a. L_3

b. R_3

c. T_3

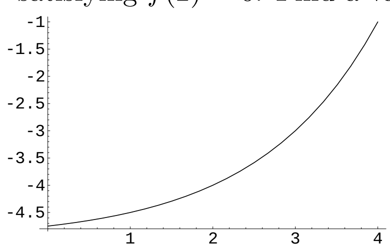
d. M_3



In each case indicate whether the estimate is an overestimate or an underestimate and explain why.

In this problem, the function I graphed was ln(x) (you didn't need to know that). The function values are approximately the following: f(1) = 0; f(1.5) = .4; f(2) = .7; f(2.5) = .9; f(3) = 1.1; f(3.5) = 1.25; f(4) = 1.4. Using these values (and the fact that the width of each of the rectangles or trapezoids is 1), we get $L_3 = (0+.7+1.1) = 1.8; R_3 = .7+1.1+1.4 = 3.2; T_3 = \frac{L_3+R_3}{2} = 2.5; M_3 = .4+.9+1.25 = 2.55$. The sum L_3 is an underestimate since the function is increasing, and R_3 is an overestimate for the same reason; T_3 is an underestimate since the function is concave down, and M_3 is an overestimate for the same reason.

(6pts.) 2. Suppose that f' is the function shown in the graph below, and suppose that F is an antiderivative of f satisfying $f(2) = 0$. Find a value of n such that $|L_n - \int_2^4 F(x)dx| \leq .01$.



This is problem 17c from section 7.2. In order to get this, you need an estimate for K_1, the maximum value for the derivative of F, which is f. Since f is a decreasing function on the interval from 2 to 4 and $f(2) = 0$, the maximum value is going to be $|f(4)|$. We can estimate $f(4)$ by using the equation $\int_2^4 f'(x)dx = f(4) - f(2) = f(4)$. We don't have an exact value for this integral, but it is certainly smaller than 10 since the rectangle of height 10 (going down to -5) will contain the area under the curve from 2 to 4. Thus, I would use $K_1 = 10$. From there, it is a matter of solving the equation $\frac{K_1(b-a)^2}{2n} \leq .01$, which implies $\frac{10(4-2)^2}{2(.01)} \leq n$, so n should be at least 2000. There are lots of other ways to do this, but they should all focus on ways to estimate K_1.

(6pts.) 3. Explain the formula $|I - L_n| \leq \frac{K_1(b-a)^2}{2n}$ (hint: a good picture will go a long way in this problem!).

I am looking for a picture something like the picture on p. 433 of the book. I want to see a picture of that together with an explanation of why are we using K_1: that boils down to the explanation that the error is contained within the triangle in the picture.