Quiz 3

Davis
M212

Name:
Pledge:

(8pts.) 1. What percentage of the earth's volume is contained between the Tropic of Cancer and the Tropic of Capricorn? (Both are approximately latitude 23). You may assume that the radius of the earth is 4000 miles.

You don't need to know the radius of the earth. The volume of the earth is $\frac{4}{3}(\pi R^3)$, where R is the radius of the earth. Each disk at height y_i has volume $\pi r_i^2 \Delta y$ where $r_i^2 + y_i^2 = R^2$, so the approximate volume between the two Tropics is $\sum_{i=1}^{n} \pi(R^2 - y_i^2)\Delta y$. Letting $n \to \infty$, we get the integral $\int_{-\frac{23}{90}R}^{\frac{23}{90}R} \pi(R^2 - y^2)dy$. This integrates to $\pi R^2 y - \pi/3 y^3 |_{-\frac{23}{90}R}^{\frac{23}{90}R} = 2(\pi R^2(\frac{23}{90}R) - \pi/3(\frac{23}{90}R)^3) = 2\pi R^3(\frac{23}{90} - \frac{(\frac{23}{90})^3}{3}) \cong \pi R^3/2$. When this is compared to the volume of the earth, $\frac{4}{3}\pi R^3$, we get a ratio of $\frac{\pi R^3/2}{\frac{4}{3}\pi R^3} = \frac{3}{8}$, so the percentage of the earth's volume between the two Tropics is 37.5%.

(8pts.) 2. Explain the formula for arc length for a general function $y = f(x)$. Make sure you explain what you are doing!

See the book or notes for this!

(4pts.) 3. Suppose a spring exerts a force of 20 pounds when it is compressed 3 inches. How much work is done in stretching the spring 12 inches?

First solve for k: $20 = k(3)$, so $k = \frac{20}{3}$. Then integrate the force over the distance: $\int_0^{12} \frac{20}{3} x dx = \frac{10}{3}x^2|_0^{12} = 480$ inch-pounds.