Quiz 4

Davis
M212

Name:
Pledge:

(5pts.) 1. Find $\lim_{n\to\infty} \Sigma_{k=1}^{n}(\frac{k^2}{n^3} + \frac{k}{n^2})$ (HINT: factor out a $\frac{1}{n}$ and see what function is left).

$\lim_{n\to\infty} \Sigma_{k=1}^{n}(\frac{k^2}{n^3} + \frac{k}{n^2}) = \lim_{n\to\infty} \Sigma_{k=1}^{n}\frac{1}{n}(\frac{k^2}{n^2} + \frac{k}{n})$. This is the right hand sum for the area under the curve $f(x) = x^2$ on the interval $[0,1]$, so the limit as $n \to \infty$ of the sum is the integral $\int_0^1 (x^2 + x)dx = (\frac{x^3}{3} + \frac{x^2}{2})|_0^1 = 1/2 + 1/3 = 5/6$.

(5pts.) 2. Show that $.9999999\ldots = .9 + .09 + .009 + .0009 + \ldots = 1$.

This is a geometric series with $a = .9$ and $r = .1$. Since $|r| < 1$, we can use the formula $\frac{a}{1-r}$ to get the sum, which is $\frac{.9}{1-.1} = \frac{.9}{.9} = 1$.

(5pts.) 3. Evaluate $\Sigma_{k=1}^{\infty} \ln \frac{k}{k+1}$.

Rewrite this as $\Sigma_{k=1}^{n}(\ln k - \ln k + 1) = (\ln 1 - \ln 2) + (\ln 2 - \ln 3) + (\ln 3 - \ln 4) + (\ln 4 - \ln 5) + \cdots + (\ln n - \ln n + 1)$. This telescopes to $S_n = \ln 1 - \ln n + 1$, and that sequence goes to $-\infty$. Thus, the series diverges.

(5pts.) 4. Find all x-values for which the following series converges: $\Sigma_{k=1}^{\infty}(\frac{5x}{2})^k$.

This is a geometric series with $r = \frac{5x}{2}$, and in order to converge we must have $|\frac{5x}{2}| < 1$, or $-\frac{2}{5} < x < \frac{2}{5}$.