Quiz 6

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M212

Name:
Pledge:

(12pts.) 1. For the following series, determine whether the series converges or diverges. If the series converges, find an upper bound on its limit. Justify your answers!

a. $\Sigma_{k=1}^{\infty} \frac{k}{k^4+k^2+1}$

Compare this to $\Sigma_{k=1}^{\infty} \frac{k}{k^4} = \Sigma_{k=1}^{\infty} \frac{1}{k^3}$ which converges ($\frac{1}{k^3}$ is bigger term by term than $\frac{k}{k^4+k^2+1}$). An upper bound on its sum is found by computing $1 + \int_1^{\infty} \frac{dx}{x^3} = \frac{3}{2}$ (the 1 in the front is because the right hand sum will not include the first term).

b. $\Sigma_{i=1}^{\infty} \frac{2^i}{3^i+1}$

Compare this to $\Sigma_{k=1}^{\infty} \frac{2}{3}^k$, which is geometric and converges ($\frac{2}{3}^k$ is bigger term by term than $\frac{2^i}{3^i+1}$). The sum of the geometric series is $\frac{\frac{2}{3}}{1-\frac{2}{3}} = 2$, and that is the upper bound.

c. $\Sigma_{j=2}^{\infty} \frac{1}{j \ln j}$

Integral test: $\int_1^{\infty} \frac{dx}{x \ln x} = \ln \ln x|_1^t = \infty$. Therefore the series diverges.

(4pts.) 2. Find the sum of the series $\Sigma_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^3}$ within .01, and argue how you know it is that close.

For converging alternating series, the error is the value of the next term in the sum, so we need to find the first place where the value of the next term is smaller than .01. This occurs for $k = 4$ since the next term is $\frac{1}{5^3} = \frac{1}{125} < .01$. Thus, we can approximate the sum of the series by $\Sigma_{k=1}^{4} \frac{(-1)^{k+1}}{k^3} = 1 - \frac{1}{8} + \frac{1}{27} - \frac{1}{64} = \frac{1549}{1728} = .8964$.

(4pts.) 3. Find the interval of convergence (endpoint behavior too!) for $\Sigma_{k=1}^{\infty} \frac{(x-1)^k}{2k}$.

Use the ratio test on this: $\frac{\frac{(x-1)^{k+1}}{2(k+1)}}{\frac{(x-1)^k}{2k}} = \frac{2k}{2k+2}(x-1) \to (x-1)$. By the ratio test, L must have absolute value less than 1, so $-1 < x-1 < 1$, or $0 < x < 2$. When $x = 0$, we have an alternating harmonic series that converges; when $x = 2$, we have a harmonic series that diverges. Thus, the interval of convergence for this series is $0 \leq x < 2$.