TEST 2

Davis | Name:
M212 | Pledge:

Show all work; unjustified answers may receive less than full credit.

(25pts.) **1.** A cylindrical tank for heating oil is placed behind your house so that the circular ends are perpendicular to the ground (the tank is lying on its side).

a. If the tank has a radius of 3 feet and a length of 5 feet, find the cubic volume of the tank.

The formula for the volume of a cylinder is $V = \pi r^2 h$ for r the radius and h the height, so the volume of the oil tank is $\pi(3)^2(5) = 45\pi$ cubic feet.

b. The tank currently contains oil up to a level of 2 feet. If the oil company comes and fills the tank up to the top and charges \$2 per cubic foot of oil, how much will you be charged for this delivery of oil? YOU DO NOT NEED TO EVALUATE THIS INTEGRAL: JUST GET IT SET UP.

If you measure y from the bottom of the tank, then the volume of a "slab" of oil at depth y_i is $(5)\Delta y(2\sqrt{9-(3-y_i)^2})$. Sum this up and let $n \rightarrow infty$, yielding the integral $\int_2^6 10\sqrt{9-(3-y_i)^2}dy$. Double this integral to get the cost of the oil.

(20pts.) **2.** A communications satellite weighs 5000 pounds, and it must orbit the earth at an altitude of 20000 miles. Newton's law of gravitation states that the gravitational force is inversely proportional to the square of the distance from the center of the object. If the radius of the earth is assumed to be 4000 miles, find the work done against gravity in lifting such a satellite into orbit.

Use the equation $F(x) = \frac{k}{x^2}$: first, figure out k. $5000 = \frac{k}{(4000)^2}$, so $k = 8 \times 10^{10}$. Now integrate $F(x)$ from 4000 to 24000, yielding work of 1.66×10^7 mile-pounds of work.

(25pts.) **3. a.** A college graduate, aged 22, wants to plan for retirement. Assuming that inflation will be 4% per year, this graduate wants to have an income stream of \$100,000$e^{.04t}$ per year from age 65 to age 90. What is the present value of this income stream (assume a current interest rate of 6%)?

$PV = \int_a^b p(t)e^{-rt}dt = \int_{43}^{68}(100000e^{.04t})e^{-.06t}dt = 100000\int_{43}^{68} e^{-.02t}dt = 100000(-50)e^{-.02t}|_{43}^{68}$
\$832, 506.53.

b. This graduate budgets M dollars per month, and plans to increase the monthly savings each year by 4% to compensate for inflation. This graduate plans to consistently contribute to the retirement fund from the 22nd birthday until the 65th birthday. This income stream is modeled as $12Me^{.04t}$: what is the present value of this income stream (again assuming a 6% interest rate)? (NOTE: the answer will contain an M).

$PV = \int_a^b p(t)e^{-rt}dt = \int_0^{43}(12Me^{.04t})e^{-.06t}dt = 12M(-50)e^{-.02t}|_0^{43} = 346.10M.$

c. Comparing parts a and b, what monthly income M does the graduate need to set aside in order to generate the retirement income?

$832,503.53 = 346.10M; M = 2405.39$. (NOTE: this is not realistic since we haven't taken into account the fact that this college grad will make intelligent investment decisions that will make the money grow rather than sticking in the cookie jar as this problem implicitly assumes!)

(20pts.) **4. a.** Approximate the integral $\int_{11}^{13} \frac{dx}{x^3+1}$ using L_2. Find a bound on the error.

$L_2 = f(11)(1) + f(12)(1) = .00133$. The error is bounded by $|f(13) - f(11)|(1) = .0003$.

b. Approximate the integral $\int_{11}^{\infty} \frac{dx}{x^3+1}$ within .01. Make sure you justify how you know how close you are to the correct answer.

We can use the area in part a. as our approximation, so that is .00133. The error will be the error from the first part (.0003) plus the error we have from ignoring the tail. The area in the tail is bounded by $\int_{13}^{\infty} \frac{dx}{x^3}$, and this integral has area .003. Thus, the total error is at most .0033, and this is less than .01 as required.

(10pts.) **5.** Find the following limits:

a. $\lim_{t\to\infty} \frac{t^2}{e^t}$

Applying L'Hopital's rule twice, we get $\frac{t^2}{e^t} \to \frac{2t}{e^t} \to \frac{2}{e^t} \to 0$ as $t \to \infty$.

b. $\lim_{x\to 0} \frac{1-x-e^{-x}}{1-\cos x}$

Applying L'Hopital's rule twice, we get $\frac{1-x-e^{-x}}{1-\cos x} \to \frac{-1+e^{-x}}{\sin x} \to \frac{-e^{-x}}{\cos x} \to \frac{-1}{1} = -1$ as $x \to 0$.