Quiz 2

Davis
M212

Name:
Pledge:

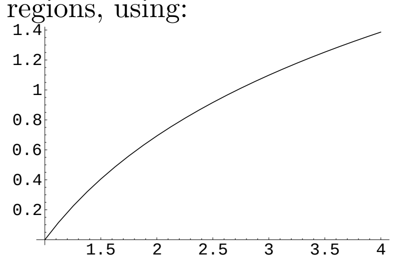
(8pts.) 1. Verify formula 47 in the Table of Integrals (a) by differentiation and (b) by using the substitution $t = a + bu$. Use that table entry to find an antiderivative for $\int \frac{3e^x}{4+5e^x} e^x dx$.

The derivative of the right hand side with respect to u is $\frac{1}{b^2}(b - \frac{ab}{a+bu}) = \frac{1}{b^2}\frac{ab+b^2u-ab}{a+bu} = \frac{u}{a+bu}$ as claimed.

The suggested substitution leads to $\frac{1}{b^2}\int \frac{(t-a)dt}{t} = \frac{1}{b^2}\int(1 - \frac{a}{t})dt = \frac{1}{b^2}(t - a\ln|t|) + C = \frac{1}{b^2}(a + bu - a\ln|a+bu|) + C$.

Using the substitution $u = e^x$, we get $\int \frac{3e^x}{4+5e^x} e^x dx = \int \frac{3u}{4+5u} du = \frac{3}{25}(4 + 5u - 4\ln|4+5u|) + C = \frac{3}{25}(4 + 5e^x - 4\ln|4+5e^x|) + C$.

(8pts.) 2. Given the picture of $y = f(x)$, estimate the area under the curve from 1 to 4, subdividing the interval into 3 regions, using:



a. T_3

b. M_3

You are told that $K_2 = 1$: how close are each of the estimates in parts a and b? How many subintervals do you need to get the estimate within .00001?

Approximately, $T_3 = 1(\frac{0+.6}{2}) + 1(\frac{.6+1}{2}) + 1(\frac{1+1.3}{2}) = .3 + .8 + 1.15 = 2.25$. Similarly, $M_3 = 1(.4) + 1(.9) + 1(1.2) = 2.5$. With the value of K_2 given, the trapezoid rule is within $\frac{1(3)^3}{12(3^2)} = .25$ and the midpoint rule is within $\frac{1(3)^3}{24(3^2)} = .125$. In order to get the Trapezoid estimate within .00001, we need to satisfy $\frac{1(3)^3}{12n^2} \leq .00001$, so $n \geq \sqrt{\frac{3^3}{12(.00001)}} = 475$ subintervals.

(4pts.) 3. You are trying to find the area under e^{-x^2} from 0 to 2 by using the midpoint rule with n subintervals. What is the area of the i^{th} rectangle?

The area of the i^{th} rectangle is $\frac{2}{n} e^{-\frac{2i-1}{n}^2}$.