Quiz 4

Davis
M212

Name:
Pledge:

(9pts.) 1. For the following series, determine whether the series converges or diverges. If the series converges, find its sum. Justify your answers!

a. $\Sigma_{n=1}^{\infty} 7\frac{(-2)^n}{5^n}$

This is a geometric series with $r = \frac{-2}{5}$, so it converges to $\frac{a}{1-r} = \frac{\frac{-14}{5}}{1-(-\frac{2}{5})} = -2$.

b. $\Sigma_{i=0}^{\infty}(e^{-i} - e^{-(i+1)})$

This series telescopes, and $S_n = e^0 - e^{-(n+1)}$. As $n \to \infty, S_n \to e^0 = 1$. Alternatively you could do this as the subtraction of two geometric series.

c. $\Sigma_{j=2}^{\infty}\frac{13}{\sqrt{j}-1}$

The comparison theorem works here, where we compare this to $\Sigma_{j=2}^{\infty}\frac{13}{\sqrt{j}}$. We can verify that this new series is smaller than the one we started with by doing a simple cross multiplication. The new series is 13 times a p-series with $p = 1/2$, so that diverges, forcing the series in question to also diverge.

(4pts.) 2. How many terms in the series $\Sigma_{n=1}^{\infty}\frac{1}{n^8}$ do we need to get accuracy within .0001?

We need $R_N < \int_N^{\infty}\frac{1}{x^8}dx < .0001$, or $\frac{1}{7N^7} < .0001$. Solving this, we see that N must be at least 2.8, so we can use 3 terms and guarantee that we have the required accuracy.

(7pts.) 3. Show that the sum of a geometric series $\Sigma_{i=0}^{\infty}ar^i$ is $\frac{a}{1-r}$ if $-1 < r < 1$ by using the sequence of partial sums.

The n^{th} partial sum is $S_n = a + ar + ar^2 + \cdots + ar^n$. We subtract rS_n from S_n to give $(1-r)S_n = a - ar^{n+1}$. Solving for S_n yields $S_n = \frac{a-ar^{n+1}}{1-r}$. If $-1 < r < 1$, then $r^{n+1} \to 0$ as $n \to \infty$, so $\lim_{n\to\infty} S_n = \frac{a}{1-r}$ as claimed.