Quiz 5

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M212

Name:
Pledge:

(9pts.) 1. Find the interval of convergence for the following series. Make sure you check the endpoints (if they exist)!

a. $\Sigma_{n=0}^{\infty} \frac{(x-1)^n}{n3^n}$

The Ratio Test yields:

$$lim_{n\to\infty}\left|\frac{\frac{(x-1)^{n+1}}{(n+1)3^{n+1}}}{\frac{(x-1)^n}{n3^n}}\right| = lim_{n\to\infty}\left|\frac{n(x-1)}{3(n+1)}\right| = \frac{|x-1|}{3} = L$$

We must have $L < 1$ for convergence, so $|x-1| < 3$, or $-2 < x < 4$. Checking the endpoints, the series we get when we substitute in $x = 4$ is the harmonic series, so it diverges, whereas the series we get when we substitute in $x = -2$ is the alternating harmonic series and hence converges. Thus, the interval of convergence for this series is $-2 \leq x < 4$.

b. $\Sigma_{k=0}^{\infty} \frac{(2k)!x^k}{k!}$

The Ratio Test yields:

$$lim_{k\to\infty}\left|\frac{\frac{(2k+2)!x^{k+1}}{(k+1)!}}{\frac{(2k)!x^k}{k!}}\right| = lim_{k\to\infty}\left|\frac{x(2k+2)(2k+1)}{k+1}\right| = \infty = L$$

The last two equalities are true unless $x = 0$, in which case $L = 0$ and the series converges. Thus, the interval of convergence is $x = 0$.

c. $\Sigma_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$

The Ratio Test yields:

$$lim_{k\to\infty}\left|\frac{\frac{x^{2(k+1)+1}}{(2(k+1)+1)!}}{\frac{x^{2k+1}}{(2k+1)!}}\right| = lim_{k\to\infty}\left|\frac{x^2}{(2k+3)(2k+2)}\right| = 0 = L$$

This is true for all values of x, so the interval of convergence is all values of x.

(5pts.) 2. Find a power series representation for the function $f(x) = \frac{1}{4+x^2}$ and determine the interval of convergence.

Rewrite this function as $f(x) = \frac{1}{4}\frac{1}{1-(-\frac{x^2}{4})}$. Substituting $-\frac{x^2}{4}$ into the geometric series yields the series $f(x) = \frac{1}{4}(1 - \frac{x^2}{4} + \frac{x^4}{16} - \frac{x^6}{64} + \cdots) = \frac{1}{4}\Sigma_{k=0}^{\infty}\frac{(-1)^k x^{2k}}{4^{k+1}}$. The series will converge as long as $\frac{x^2}{4} < 1$, or $-2 < x < 2$.

3.

(6pts.) **a.** Given the power series $\frac{1}{\sqrt{1-x}} = \Sigma_{k=0}^{\infty} \frac{(2k+1)(2k-1)\cdots 5\cdot 3\cdot 1}{2^k} x^k = 1 + \frac{1}{2}x + \frac{3}{4}x^2 + \frac{15}{8}x^3 + \frac{105}{16}x^4 + \cdots$, find at least the first 4 nonzero terms for the power series for $\frac{1}{\sqrt{1-x^2}}$.

This is a simple substitution of x^2 for x, so $\frac{1}{\sqrt{1-x^2}} = \Sigma_{k=0}^{\infty} \frac{(2k+1)(2k-1)\cdots 5\cdot 3\cdot 1}{2^k} x^{2k} = 1 + \frac{1}{2}x^2 + \frac{3}{4}x^4 + \frac{15}{8}x^6 + \frac{105}{16}x^8 + \cdots$

b. Use part a. to get at least the first 4 nonzero terms for the power series for $\sin^{-1}(x)$ (Hint: you will either have to differentiate or integrate the power series from part a. Explain your choice.).

Since $\sin^{-1}(x) = \int \frac{1}{\sqrt{1-x^2}} dx$, we need to integrate the series from part a to get the power series for $\sin^{-1}(x)$. Thus,

$$\sin^{-1}(x) = \Sigma_{k=0}^{\infty} \frac{(2k+1)(2k-1)\cdots 5\cdot 3\cdot 1}{(2k+1)2^k} x^{2k+1} = x + \frac{1}{6}x^3 + \frac{3}{20}x^5 + \frac{15}{56}x^7 + \frac{105}{144}x^9 + \cdots$$