Quiz 6

Davis
M212

Name:
Pledge:

(10pts.) 1. Find the first four nonzero terms of the Taylor Series expansion for the function $f(x) = \sin(2x)$ about the point $a = \frac{\pi}{4}$. Sketch the graph of $f(x)$ together with two of its approximations (make sure you label your graph!).

The first four terms of the Taylor Series expansion are $\sin(2x) = 1 - 2(x - \frac{\pi}{4})^2 + \frac{2}{3}(x - \frac{\pi}{4})^4 - \frac{4}{45}(x - \frac{\pi}{4})^6 + \cdots$. The period of $\sin(2x)$ is π, and the easiest two approximations to graph are 1 and $1 - 2(x - \frac{pi}{4})^2$, the latter being an upsidedown parabola touching the sine curve at its peak value at $(\pi/4, 1)$.

2.

(10pts.) **a.** Find the first 4 nonzero terms of the MacLaurin series for $\frac{1}{\sqrt{1-x}}$.

The first four terms are $1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{15}{48}x^3 + \cdots$.

b. Use the first 4 terms from the series in part a to estimate the value of $\frac{1}{\sqrt{.5}}$. What bound can we get on the error?

Plugging in $x = .5$ will yield the approximation, $1 + \frac{1}{2}(.5) + \frac{3}{8}(.5)^2 + \frac{15}{48}(.5)^3 = 1.336$. The error can be no worse than $\frac{M}{4!}(.5)^4$, where M is the bigger of $\frac{105}{16}$ and $\frac{105}{16}(.5)^{-\frac{9}{2}} \cong 150$. Thus, the error is no worse than $\frac{150}{4!}(.5)^4 = .39$. The actual value of the function at $x = .5$ is $\sqrt{2} \cong 1.414$, which is only .078 away from the value we approximated earlier.