Quiz 7

Davis
M212

Name:
Pledge:

(10pts.) 1. The population of the world was about 5.3 billion in 1990 and about 6.1 billion in 2000. Assume that the carrying capacity for world population is 50 billion. Write the logistics differential equation for this data and solve it (show your work to solve it!). What will the population be in 2050 according to this model?

The differential equation is $\frac{dP}{dt} = kP(1-\frac{P}{50})$. Separate the variables, integrate by using partial fractions, and combine the right hand side using property of logarithms to get $\ln|\frac{P}{50-P}| = kt+C$. Take e to the power of each side, multiply by $50-P$, and combine the P's to get $P(1+e^{kt}e^C) = 50e^{kt}e^C$. Solve for $P(t) = \frac{50e^{kt}e^C}{1+e^{kt}e^C}$ (note that the book has a different version of the same formula, and that was fine as well).

Using the information given in the problem, plug in $t = 0$ and $P = 5.3$ to solve for $e^C = \frac{5.3}{44.7}$. Plug in $t = 10$ and $P = 6.1$ to solve for $k = .01586$. Plug in $t = 60$ to get $P(60) = 11.7$, the population of the world in the year 2050 (in billions).

2. The Pacific halibut fishery has been modeled by the differential equation $\frac{dP}{dt} = .4P(1-\frac{P}{400}) - 30$.

(7pts.) **a.** Draw a direction field for this differential equation, and sketch the solution with the initial condition $P(0) = 120$.

The key thing for this part was to solve $\frac{dP}{dt} = 0$, which had solutions $P = 100$ and $P = 300$. If you draw horizontal lines at those levels (and another at $P = 400$, the max population), you should have the direction field increasing for $100 \leq P \leq 300$, and decreasing elsewhere. If you start at $P = 120$, the curve should look like a logistics equation starting at $P = 120$.

(3pts.) **b.** Use Euler's method with step size $h = 5$ to approximate the number of fish you have after 10 weeks starting with $P(0) = 120$?

The number of fish after 5 weeks is $120 + 5[(.4)(120)(1-\frac{120}{400}) - 30] = 138$, and the number of fish after 10 weeks is $138 + 5[(.4)(138)(1-\frac{138}{400}) - 30] \cong 169$. Many people went too far: stop at this point and state that there will roughly be 169 fish after 10 weeks.