Quiz 1

Davis
M212

Name:
Pledge:

(8pts.) 1. Use integration by parts to justify the following formula: $\int \ln^n(x)dx = x\ln^n(x) - n\int \ln^{n-1}(x)dx$. Use the formula to find an antiderivative for $\int \ln^2(x)dx$.

Choose $u = \ln^n(x), v' = 1$, hence $u' = n\ln^{n-1}(x)(\frac{1}{x}), v = x$. Integration by parts leads directly to the formula above. Applying this formula to the $n = 2$ case yields $\int \ln^2(x)dx = x(\ln^2(x)) - 2\int \ln(x)dx$, and then applying the formula with $n = 1$ yields $\int \ln^2(x)dx = x(\ln^2(x)) - 2x\ln(x) - \int \ln^0(x)dx = x(\ln^2(x)) - 2x\ln(x) - x + C$.

(12pts.) 2. Integrate the following:

a. $\int \frac{\cos(2x)}{\sin(2x)+1}dx$

Use $u = \sin(2x), du = 2\cos(2x)$ to change the integral into $\frac{1}{2}\int \frac{du}{u+1}$. We could do another u-substitution to integrate this or just recognize this as $\frac{1}{2}\ln(u+1) + C = \frac{1}{2}\ln(\sin(2x)+1) + C$.

b. $\int \sin^{-1}(t)dt$

Integration by parts using $u = \sin^{-1}(t), v' = 1, u' = \frac{1}{\sqrt{1-t^2}}, v = t$. Applying the formula yields $\int \sin^{-1}(t)dt = t\sin^{-1}(t) - \int \frac{t}{\sqrt{1-t^2}}dt$. A u-substitution in the integral leads to $\sin^{-1}(t)dt = t\sin^{-1}(t) + \sqrt{1-t^2} + C$.

c. $\int \frac{2x+7}{x^2+7x+12}dx$

This is a u-substitution, with $u = x^2+7x+12$, so the antiderivative is $\ln(x^2+7x+12) + C$. If you didn't recognize that and went through a partial fraction derivation, you would write the integrand as $\frac{2x+7}{x^2+7x+12} = \frac{A}{x+3} + \frac{B}{x+4}$. Solving for A and B leads to $A = B = 1$ and the same antiderivative.