Quiz 2

Davis
M212

Name:
Pledge:

(8pts.) 1. Estimate $\int_1^4 \ln(x)dx$ using:

a. T_3

The width of the trapezoids is 1, so we get the formula $T_3 = 1(\frac{\ln(1)+\ln(2)}{2} + \frac{\ln(2)+\ln(3)}{2} + \frac{\ln(3)+\ln(4)}{2}) = 2.485$.

b. M_3

As in part a., the width of the rectangles is 1, yielding the formula $M_3 = 1(\ln(1.5) + \ln(2.5) + \ln(3.5)) = 2.575$.

c. Draw a picture and explain from that which of T_3 and M_3 is an overestimate and which is an underestimate.

The graph of $y = \ln(x)$ is concave down. Because of this, the trapezoids will all be below the curve, so T_3 is an underestimate. Also because of the concavity, if you pivot the midpoint rectangles about the midpoint-function point until the top of the rectangle is the tangent line to the curve, we see that the area of that trapezoid (which is equal to the area of the midpoint-rectangle) is greater than the area under the curve over that interval. This implies that M_3 is an overestimate.

d. How many subintervals do you need to get the midpoint estimate within .01?

We first need to compute K by computing the second derivative of $y = \ln(x)$, which is $f''(x) = -\frac{1}{x^2}$. The maximum value of $|f''(x)|$ on the interval $[1,4]$ is $K = 1$. Plugging that into our formula, we get $.01 \geq \frac{(1)(4-1)^3}{24n^2}$, which simplifies to $n^2 \geq \frac{2700}{24}$, which implies that $n \geq 11$ works.

(4pts.) 2. Use the comparison theorem to get a bound on the area under e^{-x^2} on the interval $[2,\infty]$, and explain what you are doing.

$x \geq 2$ implies $x^2 \geq 2x$, or $-x^2 \leq -2x$ and $e^{-x^2} \leq e^{-2x}$. This helps us to see that $\int_2^\infty e^{-x^2}dx < \int_2^\infty e^{-2x}dx = \lim_{t\to\infty}[\int_2^t e^{-2x}dx] = \lim_{t\to\infty}[-\frac{1}{2}e^{-2x}|_2^t] = \lim_{t\to\infty}[-\frac{1}{2}(e^{-2t} - e^{-4})] = \frac{e^{-4}}{2} \cong .01$ is a bound on the area.

(8pts.) 3. Determine whether the following integrals converge or diverge. If it converges, determine the exact answer.

a. $\int_2^\infty \frac{dx}{\sqrt{x}-1}$

Compare this integral with $\int_2^\infty \frac{dx}{\sqrt{x}}$: you could either say that subtracting 1 from the denominator makes the denominator smaller and therefore the fraction bigger, or you could go through the following justification. $0 > -1; \sqrt{x} > \sqrt{x} - 1; \frac{1}{\sqrt{x}-1} > \frac{1}{\sqrt{x}}$. In either case, we have $\int_2^\infty \frac{dx}{\sqrt{x}-1} > \int_2^\infty \frac{dx}{\sqrt{x}} = \infty$ (this last equality comes from our work on $\int_1^\infty \frac{dx}{x^p}$, and the integral diverges for $p \leq 1$). By the comparison theorem this area is infinite.

b. $\int_1^\infty xe^{-x}dx$

$\int_1^\infty xe^{-x}dx = \lim_{t\to\infty}[\int_1^t xe^{-x}dx] = \lim_{t\to\infty}[-xe^{-x}|_1^t + \int_1^t e^{-x}dx] = \lim_{t\to\infty}[-te^{-t} + e^{-1} - e^{-t} + e^{-1}] = 2e^{-1}$. The final equality uses L'Hospital's rule to compute the integral, and the first integration uses either the table in the back of the book or integration by parts.