Quiz 3

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M212 — Pledge:

(8pts.) 1. Show that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

The volume of a slice of the sphere is $\pi s^2 \Delta y$ (I have done my slices horizontally and called the radius of the cross section s to avoid confusion with the radius r of the sphere). Since I label my y-axis starting at the center of the sphere, I can draw in a right triangle with sides s, y, and hypotenuse r. The Pythagorean theorem implies that $y^2 + s^2 = r^2$, or $s^2 = r^2 - y^2$. Our estimate for the volume of the sphere is $\Sigma\pi(r^2 - y^2)\Delta y$. When we let the number of slices go to infinity, we get that the exact area of the sphere is $\int_{-r}^{r} \pi(r^2 - y^2)dy = \frac{4}{3}\pi r^3$.

(6pts.) 2. Set up (but do not solve) the integrals required to compute the volume generated by revolving the area under $y = \sin(x)$, above the x-axis, and between $x = 0$ and $x = \pi$:

a about the line $y = -1$

We do this the washer method (the washers are vertical), so the volume of a washer is $(\pi R^2 - \pi r^2)\Delta x$, where $R = \sin(x) - (-1)$ and $r = 0 - (-1)$. Our estimate for the volume of the object is $\Sigma\pi((\sin(x)+1)^2 - 1)\Delta x$, and the exact area is found by letting the number of washers go to infinity: $\int_0^{\pi} \pi((\sin(x) + 1)^2 - 1)dx$

b about the line $x = -\pi$.

Show your work (don't just write down the formula!).

This is much easier to do the shell method. The volume of a shell is $2\pi r h\Delta x$, where $r = x - (-\pi)$ and $h = \sin(x)$. Our estimate for the volume of the object is $\Sigma 2\pi(x+\pi)\sin(x)\Delta x$, and when we let the number of shells go to infinity we get the exact volume of $\int_0^{\pi} 2\pi(x+\pi)\sin(x)dx$.

(6pts.) 3. My basement flooded during the Hurricane Isabel. The floor of the basement is about 4 feet below ground level, and I had 0.5 feet of water covering a rectangular basement that is 40 feet by 25 feet. Assuming that water has a density of 62.5 pounds per cubic foot, how much work was required to bail the water out of my basement (the pump that usually does this work doesn't work when we don't have electricity :-()?

The volume of a slice of water is $1000\Delta y$, and the weight of that slice is $(62.5)1000\Delta y$ (this is the force). We need to move this $(4 - y)$ to get it out of the basement, so the approximation for the work done is $\Sigma(62.5)1000\Delta y(4 - y)$. When we let the number of slices go to infinity we get $\int_0^{.5} 62500(4 - y)dy = 250000y - 31250y^2|_0^{.5} = 125000 - 7812.5 = 117187.5$ foot-pounds of work.