Quiz 4

Davis
M212

Name:
Pledge:

(9pts.) 1. For the following series, determine whether the series converges or diverges. If the series converges, find its sum. Justify your answers!

a. $\Sigma_{n=1}^{\infty} 5\frac{(-7)^n}{2^n}$

This is a geometric series with $r = \frac{-7}{2} < -1$, so the series diverges. You could also have used either the Ratio Test (where you get $L = \frac{-7}{2}$) or the n^{th} term divergence test (the limit of the terms is infinite).

b. $\Sigma_{i=1}^{\infty}(\pi^{-i} - \pi^{-(i+1)})$

This series converges either by telescoping or geometric. If you do telescoping, then the n^{th} partial sum is $S_n = \frac{1}{\pi} - \frac{1}{\pi^{n+1}}$, which goes to $\frac{1}{\pi}$ as $n \to \infty$.

c. $\Sigma_{j=2}^{\infty} \frac{(-1)^j j^2}{5j^2+3}$

This diverges by the n^{th} term divergence test (or the alternating series test) since the limit of the terms is $\frac{1}{5} \neq 0$.

(4pts.) 2. How many terms in the series $\Sigma_{n=1}^{\infty} \frac{1}{n^4}$ do we need to get accuracy within .001?

We need $R_N < \int_N^{\infty} \frac{1}{x^4} dx < .001$, or $\frac{1}{3N^3} < .001$. Solving this, we see that N must be at least 6.9, so we can use 7 terms and guarantee that we have the required accuracy.

(7pts.) 3. Show that the sum of a geometric series $\Sigma_{i=0}^{\infty} ar^i$ is $\frac{a}{1-r}$ if $-1 < r < 1$ by using the sequence of partial sums.

The n^{th} partial sum is $S_n = a + ar + ar^2 + \cdots + ar^n$. We subtract rS_n from S_n to give $(1-r)S_n = a - ar^{n+1}$. Solving for S_n yields $S_n = \frac{a - ar^{n+1}}{1-r}$. If $-1 < r < 1$, then $r^{n+1} \to 0$ as $n \to \infty$, so $\lim_{n\to\infty} S_n = \frac{a}{1-r}$ as claimed.