Quiz 5

Davis
M212

Name:
Pledge:

1.

(10pts.) **a.** Find the first four nonzero terms of the MacLaurin series for $\frac{1}{\sqrt{1-x}}$.

Take the first three derivatives, yielding: $f(x) = (1-x)^{-\frac{1}{2}}$; $f'(x) = \frac{1}{2}(1-x)^{-\frac{3}{2}}$; $f''(x) = \frac{1}{2}\frac{3}{2}(1-x)^{-\frac{5}{2}}$; $f'''(x) = \frac{1}{2}\frac{3}{2}\frac{5}{2}(1-x)^{-\frac{7}{2}}$; Plug 0 into each of those, yielding: $f(0) = 1$; $f'(0) = \frac{1}{2}$; $f''(0) = \frac{3}{4}$; $f'''(0) = \frac{15}{8}$. Thus, the MacLaurin series is $\frac{1}{\sqrt{1-x}} = 1 + \frac{1}{2}x + \frac{\frac{3}{4}}{2!}x^2 + \frac{\frac{15}{8}}{3!}x^3 + \cdots$.

b. Use your answer to part a. to estimate $\frac{1}{\sqrt{.9}}$ within .01.

If this were alternating (which is what I intended), then you would only need to add the first two terms giving an estimate of 1.05 (the actual answer is 1.054). As it is stated, you need the remainder formula from the Taylor expansion and some estimate for M, the maximum value of the second derivative, to justify the answer. Best to think of this as alternating...

2.

(10pts.) **a.** Verify that $P(t) = \frac{K}{1+Ae^{-kt}}$ is a solution to the differential equation $\frac{dP}{dt} = kP(1 - \frac{P}{K})$.

First take the derivative, giving $P'(t) = -K(1 + Ae^{-kt})^{-2}Ae^{-kt}(-k)$. Then plug $P = \frac{K}{1+Ae^{-kt}}$ into the right hand side of the differential equation, giving $k\frac{K}{1+Ae^{-kt}}(1 - \frac{\frac{K}{1+Ae^{-kt}}}{K}) = \frac{kK}{1+Ae^{-kt}}(\frac{1+Ae^{-kt}}{1+Ae^{-kt}} - \frac{1}{1+Ae^{-kt}}) = \frac{kKAe^{-kt}}{(1+Ae^{-kt})^2}$. Comparison demonstrates that $P = \frac{K}{1+Ae^{-kt}}$ is a solution to the given differential equation.

b. Sketch the direction field for $\frac{dP}{dt} = P(1 - \frac{P}{4})$. Use your direction field to sketch a solution curve passing through the point $(0, 1)$.

The direction field is the same as the ones we drew in class. The critical feature is the equilibrium solutions at $P = 0$ and $P = 4$. The solution curve is an s-curve starting at $(0, 1)$ and approaching $P = 4$ as $t \to \infty$.