Quiz 1

Davis
M211

Name:
Pledge:

(8pts.) 1. Use integration by parts to justify the following formula: $\int \sin^n(x)dx = -\frac{1}{n}\sin^{n-1}(x)\cos(x) + \frac{n-1}{n}\int \sin^{n-2}(x)dx$ for n positive.

Choose $u = \sin^{n-1}(x), dv = \sin(x)dx$: that implies that $du = (n-1)\sin^{n-2}(x)\cos(x)$ and $v = -\cos(x)$. Applying parts we get $\int \sin^n(x)dx = -\sin^{n-1}(x)\cos(x) + (n-1)\int \sin^{n-2}(x)\cos^2(x)dx$. Substituting $\cos^2(x) = 1 - \sin^2(x)$, we get $\int \sin^n(x)dx = -\sin^{n-1}(x)\cos(x) + (n-1)\int \sin^{n-2}(x)dx - (n-1)\int \sin^n(x)dx$. Adding the last term to the other side and dividing by n yields the answer.

(8pts.) 2. The Environmental Protection Agency was recently asked to investigate a spill of radioactive iodine. The rate of emission of radioactive iodine satisfies $R(t) = R_0 e^{-.004t}$, where $R(t)$ is measured in millirems per hour and R_0 is the initial rate.

a. If the initial rate at the time of the spill is 5 *milli*rems per hour and serious illness occurs when the exposure reaches 100 millirems, how long do they have to evacuate everyone from the affected area?

b. What is the average exposure rate over the first 100 hours? When does the actual exposure rate equal the average?

a. $\int_0^x 5e^{-.004t}dt = 100$; $e^{-.004x} - 1 = -.08$ (with a u-substitution). Thus, $x = \ln .92/-.004 = 20.8$ hours before people will become ill.

b. $\frac{\int_0^{100} 5e^{-.004t}dt}{100-0} = \frac{1}{-.08}(e^{-.4} - 1) = 4.12$. This rate is achieved when $5e^{-.004t} = 4.12$, or $t = 48.3$ hours.

(4pts.) 3. *WITHOUT YOUR CALCULATOR*, do the following:

a. $\int \sin(e^{2t})e^{2t}dt$

b. $\int_0^{\frac{\pi}{2}} \cos^2(\theta)\sin(\theta)d\theta$

a. Use $u = e^{2t}$ to get $\int \sin(e^{2t})e^{2t}dt = -\frac{1}{2}\cos(e^{2t}) + C$.

b. Use $u = \cos(\theta)$ to get $-\frac{1}{3}\cos^3(\theta)|_0^{\frac{\pi}{2}} = 0 - (-\frac{1}{3}) = \frac{1}{3}$.