Quiz 3

Davis
M211

Name:
Pledge:

(10pts.) 1. Consider a rod of length 1, with density $\delta(x) = 1 + kx$, where k is a positive constant. Suppose the rod is lying on the positive x-axis with one end at the origin.

a. Find the center of mass as a function of k.

$$\overline{x} = \frac{\int_0^1 x(1+kx)dx}{\int_0^1 (1+kx)dx} = \frac{\frac{1}{2}+\frac{k}{3}}{1+\frac{k}{2}}$$

b. How far to the right can the center of mass be?

The formula for center of mass above increases as k gets bigger, and the limit as $k \to \infty$ is $\frac{2}{3}$, so the center of mass cannot be any further right than $\frac{2}{3}$.

(10pts.) 2. You are digging a hole for a rectangular swimming pool 20 feet by 10 feet by 5 feet deep. The dirt is more compact the deeper you go, and the density is $\delta(x) = x + 5$ pounds per cubic foot when you are x feet below ground level. How much work do you need to do to dig this hole?

Take a "slab" of dirt at depth x and width Δx: it has volume $20(10)\Delta x$ and weight $20(10)\Delta x(x + 5)$. This slab is moved x feet up (measuring x from ground level), so the work done on this slab is $20(10)\Delta x(x + 5)x$. When we add the other slabs, we get an approximation of work done as $\sum_i 20(10)\Delta x(x_i + 5)x_i$ (I have put the subscript i in the equation to indicate that the slabs come from different depths). Letting the number of intervals go to ∞ turns this into an integral, $\int_0^5 20(10)x(x + 5)dx = 20,833.3$ foot-pounds of work.

(YOU MAY IGNORE THIS LAST PART IF YOU DON'T WANT TO WORRY ABOUT MEASURING THE DISTANCE FROM THE BOTTOM OF THE POOL!)

If you measured distance from the bottom of the pool, I think it is best to introduce a different variable than x to measure this so you avoid confusion with the variable in the density function (I will use h). The volume of a slab at height h is still $20(10)\Delta h$ (note that I have changed the width of the slab to Δh). The weight per cubic foot at height h is the same as the weight per cubic foot at depth $5 - h$, and we know the formula for the weight per cubic foot at depth x is $\delta(x) = x + 5$. Thus, in terms of h, the weight per cubic foot at depth $5 - h$ is $(5 - h) + 5 = 10 - h$. Multiplying by volume yields $20(10)\Delta h(10 - h)$ pounds for the slab. This slab is moved a distance of $5 - h$, yielding work of $20(10)\Delta h(10 - h)(5 - h)$. We turn this into a Riemann sum and then an integral as before, and we get the integral $\int_0^5 20(10)(10 - h)(5 - h)dh = 20,833.3$ foot-pounds of work.