Last Quiz!!!!!

Davis
M212

Name:
Pledge:

(11pts.) 1. Estimate the magnitude of the error in approximating $\frac{1}{(1+.2)^2}$ using the second degree Taylor polynomial for $f(x) = \frac{1}{(1+x)^2}$ about $x = 0$. What is the actual error?

First figure out M, the maximum value of the third derivative on the interval [0,.2]. In this case, the third derivative is $24(1+x)^{-5}$, and this has a max value at $x = 0$ of $M = 24$ (in absolute value). Plugging this into the formula, we get that the error in using the second degree Taylor polynomial is at most $\frac{24}{3!}(.2)^3 = .032$. The actual error comes from comparing the actual value of $\frac{1}{(1+.2)^2} = .694444$ with the Taylor polynomial estimate of $1 - 2(.2) + 3(.2)^2 = .72$ to get $E_2(.2) = .72 - .694444 = .0255555$.

(9pts.) 2. Determine whether the following series converge, and explain your reasoning.

a. $\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$

Use the ratio test for this: $\lim_{n\to\infty} \frac{\frac{(n+1)!}{(2(n+1))!}}{\frac{n!}{(2n)!}} = \lim_{n\to\infty} \frac{n+1}{(2n+2)(2n+1)} = 0 = L$, which is less than 1, so the series converges.

b. $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$

Use the integral test: $\int_2^{\infty} \frac{1}{x \ln(x)} dx = \lim_{b\to\infty} \ln(\ln(x))|_2^b = \lim_{b\to\infty} \ln(\ln(b)) - \ln(\ln(2)) = \infty$, so the series diverges.

c. $\sum_{n=1}^{\infty} \frac{2}{n+1}$

Use either the integral test or the comparison test. Integral test: $\int_1^{\infty} \frac{2}{x+1} dx = \lim_{b\to\infty} 2\ln(x+1)|_1^b = \lim_{b\to\infty} 2\ln(b+1) - 2\ln(2) = \infty$, so the series diverges. Comparison test: since $n \geq 1$, we add n to both sides to get $2n \geq n+1$, so $\frac{2}{n+1} \geq \frac{1}{n} \geq 0$. Since the harmonic series $\sum \frac{1}{n}$ diverges, that implies by the comparison test that $\sum_{n=1}^{\infty} \frac{2}{n+1}$ also diverges.