$\underline{\text{TEST 1}}$

Davis	Name:
M212	Pledge:

Show all work; unjustified answers may receive less than full credit.

(20pts.) 1. Without your calculators (but possibly using the tables), compute the following:

- a. $\int_{1}^{2} xe^{x^{2}} dx$ $1/2(e^{4} e)$ b. $\int \arcsin(5x) dx$ $x \arcsin(5x) + 1/2(1 25x^{2})^{1/2} + C$ c. $\int e^{2t} \cos(3t) dt$ $1/13e^{2t}(3\sin(2t) + 2\cos(3t)) + C$ d. $\int \frac{x+3}{x^{2}+4x+5} dx$ $1/2 \ln(x^{2} + 4x + 5) + \arctan(x + 2) + C$
- (20pts.) 2. For the function pictured below, suppose we are trying to estimate $\int_0^2 f(x) dx$. Determine whether the following statements are true or false and give a one sentence justification:
 - **a.** $Right(n) \leq Trap(n)$ False: Right is overestimate, Trap is underestimate.
 - **b.** $Trap(n) \leq Simp(n)$ True: Simp is average of Trap and Mid, and Trap is an underestimate, so Simp is bigger.
 - c. $Mid(n) \leq Left(n)$ False: Mid is overestimate, Left is underestimate.
 - **d.** $\int_0^2 f(x) dx \leq Left(n)$ False: Left is underestimate of exact value.
 - e. $Right(1000) \leq Right(2000)$ False: Right(2000) is less of an overestimate than Right(1000) (it is more accurate).
- (10pts.) **3.** For the function from problem 2, estimate Left(4), state whether it is an overestimate or underestimate, and give a rough estimate of how far it is from the actual answer.

Left(4) = (1/2)(1 + 1.02 + 1.03 + 1.035) = 2.0225. This is an underestimate, and the fact that Right(4) = 2.0425 we see that it is at worst .02 away from the true answer (Right is an overestimate, Left is an underestimate, and the true answer is in between them).

(12pts.) 4. Do the following integrals converge or diverge? Explain your answer.

a.
$$\int_{5}^{\infty} \frac{2-\sin(z)}{z^{\frac{2}{3}}} dz$$

Diverges because the integral has more area than the area under $z^{-2/3}$ from 1 to ∞ , and that curve has an infinite area.

b.
$$\int_3^5 \frac{1}{(x-3)^{\frac{2}{3}}} dx$$

Converges, $3(2)^{1/3}$.

(10pts.) 5. Find the volume of a cone whose height is 3 cm and whose base radius is 1 cm.

Draw a picture of a slice of the cone with cross-sectional area a circle of radius r at height h from the base. Using similar triangles, you can see that the volume of the slice is $\pi(\frac{3-h}{3})^2\delta h$. Turning this into an integral you get $\int_0^3 \pi(\frac{3-h}{3})^2 dh = \pi$.

- (20pts.) 6. Verify that $\int x^n \ln (x) dx = \frac{1}{n+1} x^{n+1} \ln (x) \frac{1}{(n+1)^2} x^{n+1} + C, n \neq -1$ using integration by parts. Let $u = \ln (x)$ and $dV = x^n dx$: then du = 1/x and $V = \frac{x^{n+1}}{n+1}$. Applying parts, we get $\int x^n \ln (x) dx = \ln (x) \frac{x^{n+1}}{n+1} - \int \frac{x^{n+1}}{n+1} (1/x) dx$. This last part integrates to $\frac{x^{n+1}}{(n+1)^2}$ as required.
- (8pts.) **7.** Evaluate exactly $\lim_{n\to\infty} \sum_{i=1}^{n} (1+\frac{2i}{n})^2 \frac{2}{n}$ without using your calculator (hint: interpret the sum as a Right hand sum and rewrite this as an integral).

 $\lim_{n \to \infty} \sum_{i=1}^{n} \left(1 + \frac{2i}{n}\right)^2 \frac{2}{n} = \int_0^2 (1+x)^2 dx = \frac{26}{3}.$