Quiz 1

Davis
M212

Name:
Pledge:

(9pts.) 1. Use integration by parts to justify the following formula: $\int \sin^n(x)dx = -\frac{1}{n}\sin^{n-1}(x)\cos(x) + \frac{n-1}{n}\int \sin^{n-2}(x)dx$ for n positive. Use the formula to calculate $\int \sin^5(x)dx$.

$u = \sin^{n-1}(x); dv = \sin(x)$
$du = (n-1)\sin^{n-2}(x)\cos(x)dx; v = -\cos(x)$

$$\begin{aligned}
\int \sin^n(x)dx &= -\sin^{n-1}(x)\cos(x) + \int (n-1)\sin^{n-2}(x)\cos(x)(\cos(x))dx \\
&= -\sin^{n-1}(x)\cos(x) + \int (n-1)\sin^{n-2}(x)\cos^2(x)dx \\
&= -\sin^{n-1}(x)\cos(x) + \int (n-1)\sin^{n-2}(x)(1-\sin^2(x))dx \\
&= -\sin^{n-1}(x)\cos(x) + (n-1)\int \sin^{n-2}(x)dx - (n-1)\int \sin^n(x)dx
\end{aligned}$$

Adding $(n-1)\int \sin^n(x)dx$ to the both sides, we get $n\int \sin^n(x) = -\sin^{n-1}(x)\cos(x) + (n-1)\int \sin^{n-2}(x)dx$, so $\int \sin^n(x) = -\frac{1}{n}\sin^{n-1}(x)\cos(x) + \frac{n-1}{n}\int \sin^{n-2}(x)dx$ as claimed.

Applying this to the case where $n=5$, we get $\int \sin^5(x)dx =$
$-\frac{1}{5}\sin^4(x)\cos(x) + \frac{4}{5}(-\frac{1}{3}\sin^2(x)\cos(x) + \frac{2}{3}(-\cos(x)))$.

(6pts.) 2. Integrate the following:

a. $\int \sin(e^{2t})e^{2t}dt$

Setting $u = e^{2t}$, we get $du = 2e^{2t}dt$, so $\int \sin(e^{2t})e^{2t}dt = \frac{1}{2}\int \sin(u)du = -\frac{1}{2}\cos(u) + C = -\frac{1}{2}\cos(e^{2t}) + C$.

b. $\int \frac{t+2}{t^2+4t-6}dt$

Setting $u = t^2+4t-6$, we get $du = (2t+4)dt$. Multiplying by 2 on the inside and $\frac{1}{2}$ on the outside of the integral changes it to $\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|t^2+4t-6| + C$.

(5pts.) 3. Verify the following antiderivative.

$\int \arctan x dx = x\arctan x - \frac{1}{2}\ln(1+x^2) + C$

You can verify this by taking the derivative of the right hand side or doing integration by parts on the left hand side (either is acceptable). If you integrate by parts, choose $u = \arctan x, v' = 1$, which leads to $u' = \frac{1}{1+x^2}, v = x$, and $\int \arctan x dx = x\arctan x - \int \frac{x}{1+x^2}dx$. This integral can be found by doing a u-substitution with $u = 1+x^2$, leading to the formula above.