Quiz 2

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M212 — Pledge:

(6pts.) 1. Use the table of integrals at the back of the book to evaluate the following integrals:

a. $\int \frac{5}{x^2+6x+18}dx$

You need to complete the square in the denominator and substitute $u = x + 3$. This integral becomes number 17 in the back of the book, and the answer is $\frac{5}{3}Arctan(\frac{x+3}{3}) + C$.

b. $\int e^x\sqrt{2e^x - e^{2x}}e^x dx$

Use the substitution $u = e^x, du = e^x dx$ to change the integral into $\int u\sqrt{2u - u^2}du$. This is of the form of 114 in the tables with $a = 1$, so the antiderivative is (once we plug back in for x): $\frac{2e^{2x}-e^x-3}{6}\sqrt{2e^x - e^{2x}} + \frac{1}{2}Arccos(1 - e^x) + C$.

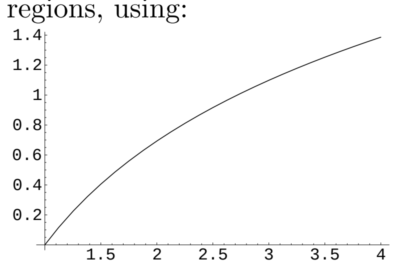
(8pts.) 2. Given the picture of $y = f(x)$, estimate the area under the curve from 1 to 4, subdividing the interval into 3 regions, using:

a. L_3

b. R_3

c. T_3

d. M_3



In each case indicate whether the estimate is an overestimate or an underestimate and explain why.

In this problem, the function I graphed was ln(x) (you didn't need to know that). The function values are approximately the following: f(1) = 0; f(1.5) = .4; f(2) = .7; f(2.5) = .9; f(3) = 1.1; f(3.5) = 1.25; f(4) = 1.4. Using these values (and the fact that the width of each of the rectangles or trapezoids is 1), we get $L_3 = (0+.7+1.1) = 1.8; R_3 = .7+1.1+1.4 = 3.2; T_3 = \frac{L_3+R_3}{2} = 2.5; M_3 = .4+.9+1.25 = 2.55$. The sum L_3 is an underestimate since the function is increasing, and R_3 is an overestimate for the same reason; T_3 is an underestimate since the function is concave down, and M_3 is an overestimate for the same reason.

(6pts.) 3. Explain the error formula $|E_L| \leq \frac{K(b-a)^2}{2n}$, where $K \geq max_{[a,b]}|f'(x)|$ (hint: a good picture will go a long way in this problem!). This error estimate puts an upper bound on the error you make by using L_n in estimating $\int_a^b f(x)dx$.

I want to see a picture of a triangle in each interval together with an explanation of why are we using K: that boils down to the explanation that the error of using L_n for the area is contained within the triangle in the picture. A proper explanations would sound something like this: "If we draw triangles in each region with the slope of the hypotenuse determined by the maximum value of the first derivative on the interval $[a, b]$, then the area contained in the triangle will cover the error in that interval. The width of each triangle is $\frac{b-a}{n}$, and the height is $K\frac{b-a}{n}$ (since the ratio of rise/run must be K), so the area of the triangle is $\frac{1}{2}(base)(height) = \frac{1}{2}\frac{b-a}{n}K\frac{b-a}{n}$. There are n triangles, so the total error is less than $n\frac{1}{2}\frac{b-a}{n}K\frac{b-a}{n} = \frac{K(b-a)^2}{2n}$ as claimed."