Quiz 3

Davis

M212

Name:

Pledge:

(8pts.) 1. What percentage of the earth's volume is contained between the Tropic of Cancer and the Tropic of Capricorn? (Both are approximately latitude 23). You may assume that the radius of the earth is 4000 miles and that the vertical distance to these lines is 1000 miles from the equator.

The volume of the earth is $\frac{4}{3}(\pi(4000)^3)$. A disk at height y has volume $\pi r^2 \Delta y$ where $r^2 + y^2 = (4000)^2$, so the volume of a slice is $\pi((4000)^2 - y^2)\Delta y$. This leads to the integral $\int_{-1000}^{1000} \pi((4000)^2 - y^2)dy$. This integrates to $\pi(4000)^2 y - \pi/3 y^3|_{-1000}^{1000} = 2(\pi(4000)^2(1000) - \pi/3(1000)^3) = 2\pi R^2(1000 - \frac{(1000)^3}{3}) \cong \pi R^3/2$. When these numbers are crunched, we get that the percentage in this region is 36.7%.

(8pts.) 2. The following formulas come from computations to determine the volume obtained from revolving an area around an axis. In each case, describe the volume being computed.

a. $\int_0^1 2\pi(2-x)(x-x^2)dx$

b. $\int_0^1 \pi((2-x^2)^2 - (2-x^{\frac{1}{3}})^2)dx$

The first integral is by the shell method, revolving the area between $y = x$ and $y = x^2$ between $x = 0$ and $x = 1$ around $x = 2$. The second integral is a washer method, revolving the area between $y = x^2$ and $y = x^{\frac{1}{3}}$ between $x = 0$ and $x = 1$ around $y = 2$.

(4pts.) 3. How much work is done in lifting a 100 pound weight 200 feet using chain that weighs 2 pounds per foot?

The function for Force is $F(x) = 100 + 2x$, where x represents the length of chain hanging over the side. If we multiply this by Δx, we get a small amount of work. Turn this into an integral, and we get $\int_0^{200}(100 + 2x)ds = 100x + x^2|_0^{200} = 20000 + 40000 = 60000 ft-lbs$ of work.