TEST 1

Davis
M212

Name:
Pledge:

Show all work; unjustified answers may receive less than full credit.

(25pts.) **1.** Without your calculators, compute the following:

a. $\int_1^2 xe^{x^2}dx$

u-substitution with $u = x^2; du = 2xdx; \frac{1}{2}\int_{u=1}^{u=4} e^u du = \frac{1}{2}(e^4 - e^1)$.

b. $\int \arcsin(5x)dx$

Integration by parts with $u = \arcsin(5x); dv = dx; du = \frac{5}{\sqrt{1-25x^2}}; v = x; \int \arcsin(5x)dx = x\arcsin(5x) - \int \frac{5x}{\sqrt{1-25x^2}}dx$; a u-substitution on this last part yields the answer of $x\arcsin(5x) - \frac{1}{5}\sqrt{1-25x^2} + C$.

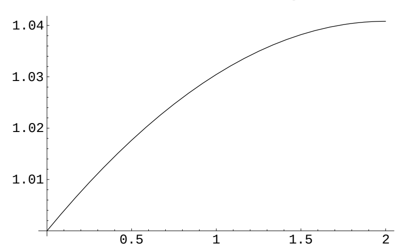
c. $\int_1^4 \frac{\ln(t)}{(t-2)^2}dt$

The first thing to recognize is that this function has a vertical asymptote at $t = 2$. If we break the integral up around that and integrate first from 1 to 2, we can use integration by parts to get $lim_{s\to 2^-} - \frac{\ln(t)}{t-2}|_1^s + \ldots$. Since this limit is infinite, we get that the integral diverges.

d. $\int \frac{x+3}{x^2+6x+5}dx$

Partial fractions. The denominator factors as $(x+5)(x+1)$, so we get two equations $A + B = 1; A + 5B = 3; B = \frac{1}{2}; A = \frac{1}{2}$, so $\int \frac{x+3}{x^2+6x+5}dx = \frac{1}{2}\ln(x+1) + \frac{1}{2}\ln(x+1) + C$.

(15pts.) **2.** For the function pictured below, suppose we are trying to estimate $\int_0^2 f(x)dx$. Determine whether the following statements are true or false and give a one sentence justification:



a. $R_n \leq T_n$

R_n is an overestimate since the function is increasing and T_n is an underestimate since the function is concave down, so this is false.

b. $T_n \leq 1.8$

T_n will be at least as big as $L_1 = 2$, so this is false.

c. $M_n \leq R_n$

M_n is an overestimate since the function is concave down and R_n is also an overestimate since the function is decreasing, but M_n is more accurate, so this is true.

d. $\int_0^2 f(x)dx \leq L_n$

L_n is an underestimate of the true answer since the function is increasing, so this is false.

e. $R_{1000} \leq R_{2000}$

R_n is an overestimate, and the bigger n gets, the more accurate the estimation, so R_{2000} is smaller than R_{1000}, so this is false.

(15pts.) **3.** Use the comparison theorem to determine whether the following integrals converge or diverge.

a. $\int_1^\infty \frac{1}{x^2+x+1}dx$

Compare this function with either $\frac{1}{x^2}$ or $\frac{1}{x^2+1}$. Both of these functions are on top of $\frac{1}{x^2+x+1}$ and both converge on the integral from 1 to ∞, so $\int_1^\infty \frac{1}{x^2+x+1}dx$ converges by the comparison theorem.

b. $\int_1^\infty \frac{1}{x^{1/2}+e^{3x}}dx$

Compare this function with $\frac{1}{e^{3x}}$. Since our function has $x^{1/2}$ added to the denominator, that makes the denominator bigger and hence the fraction smaller, so $\frac{1}{x^{1/2}+e^{3x}} \leq \frac{1}{e^{3x}}$. We integrate $\frac{1}{e^{3x}}$ and see that it converges, so by the comparison theorem $\int_1^\infty \frac{1}{x^{1/2}+e^{3x}}dx$ converges as well.

(20pts.) **4.** Verify the formula $\int u^n \sin(u)du = -u^n \cos(u) + n\int u^{n-1}\cos(u)du$ (a) by differentiation; and (b) by using integration by parts. Use the formula and the tables in the back of the book to calculate $\int u^2 \sin(u)du$.

(a) The derivative of the right hand side is $-u^n(-\sin(u)) + (-nu^{n-1})\cos(u) + nu^{n-1}\cos(u)$ (the last term comes from the fact that the derivative of an integral is the function you are integrating). This cancels to give $-u^n \sin(u)$ as expected.

(b) Choose $U = u^n, V' = \sin(u)$, which leads to $U' = nu^{n-1}$ and $V = -\cos(u)$. Integration by parts implies that $\int u^n \sin(u)du = -u^n \cos(u) + n\int u^{n-1}\cos(u)du$ as expected.

Finally, $u^2 \sin(u)du = -u^2\cos(u) + 2\int u\cos(u)du$, and this last integral can be solved using Formula 83 in the back of the book, yielding $u^2\sin(u)du = -u^2\cos(u) + \cos(u) + u\sin(u) + C$.

(20pts.) **5.** You are sitting still at a stop light, and the instant the light turns green you hit the accelerator. Your velocity at time t satisfies $v = t^2$, where v is measured in meters per second and t is measured in seconds. Just as the light turns green another car passes you going a steady 9 meters per second. When is this other car as far ahead of you as it ever will be? How far ahead is it? Approximately when do you catch up to this car? Make sure you use integration to answer this question (A good description of how you would solve this question will get most of the credit)!

The other car is as far ahead of you as it will ever be when your speeds are equal, which occurs at $t = 3$. It will be $\int_0^3 (9 - t^2)dt = 18$ meters ahead at that point. You will catch up to this car when $\int_3^s (t^2 - 9)dt = 18$, or when $s^3/3 - 9s = 0$. This occurs either when $s = 0$ (in other words at the very start) or at $s = \sqrt{27}$, and this latter time is when you catch up (slightly more than 5 seconds).