Test 1

Davis   Name:

CS222   Pledge:

(12pts.)   1. Use truth tables to determine whether the following statements are equiv-
alent (make sure to state whether they are equivalent or not):

a. $\overline{p \rightarrow q}; p \wedge \overline{q}$

b. $(p \rightarrow q) \rightarrow r; p \rightarrow (q \rightarrow r)$.

c. $(p \rightarrow q) \vee r; (r \vee q) \wedge p$

(12pts.)   2. Consider the statement $P(x, y, z) : x^2 + y^2 - z^2 \leq 36$. Which of the
following are true (justify your answer).

a. $\forall x \forall y \exists z (P(x, y, z))$

b. $\forall x \exists y \exists z (P(x, y, z))$

c. $\exists x \forall y \forall z (P(x, y, z))$

(13pts.)   3. Prove the following statement: If x and y are rational numbers, then $x+y$
is also rational.

(13pts.)   4. Prove the following statement by contradiction: For all x and y in the
real numbers, if $x^2 + y^2 = 9$, then $x \leq 3$ and $y \leq 3$.

(13pts.)   5. Prove the following: $\forall n \geq 1, \sum_{i=1}^{n}(1/2^i) = 1 - (1/2)^n$.

(13pts.)   6. The Fibonacci sequence is defined as follows: 1,1,2,3,5,8,13,21,.... The
n^{th} term of the sequence f_n is equal to the sum of the previous two terms,
so $f_n = f_{n-1} + f_{n-2}$. Prove the $f_n < 2^n$ for all $n \geq 1$.

(12pts.)   7. Determine whether the following set equations are true or not. You may
use Venn diagrams or actual sets, but make sure you label everything clearly!

a. $A - (B - C) = (A - B) - C$

b. $A \cup (B \cap C) = (A \cup B) \cap C$

(12pts.)   8. Consider the relation $R = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$ on $\{1, 2, 3, 4\}$.

a. Is this relation reflexive? symmetric? anti-symmetric? transitive? In
each case, either explain your answer or give a counterexample.

b. Draw the digraph of the relation.