Math 350
Spring, 2000

HOMEWORK #3

Do 50 points of the following problems (due 2/3/00).

25 pts. **1** Find a necessary condition on the length n so that the binary $(n, M, 3)$ code is perfect. What are the conditions for a perfect q-ary $(n, M, 3)$ code?

25 pts. **2** Let $a, b \in Z_p$ for p a prime: show that $(a + b)^p \equiv a^p + b^p \bmod p$. Explain how that can be extended to $(a+b+\cdots+z)^p \equiv a^p+b^p+\cdots+z^p \bmod p$. Use this to show that $x^p \equiv x \bmod p$ for every $x \in Z_p$.

25 pts., ★ **3** Consider the following matrix: $H = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$.

Show that the set of vectors $u = (u_1, u_2, \ldots, u_7)$ that satisfy $Hu^T = (000)$ form a binary linear code. How many elements are there in this code? Use properties of the matrix H to determine the minimum distance of the code (don't just use brute force).

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