

Math 350  
Spring, 2000

### HOMEWORK #6

Do 50 points of the following problems (due 2/24/00??).

- 10 pts.    **1** Prove that all  $q$ -ary linear Hamming Codes of a given length are equivalent.
- 20 pts.    **2** Construct  $H(5,3)$ .
- 10 pts.    **3** Use the decoding procedure that we discussed in class to decode 1001001, 0101101, and 1110000 (in  $\text{Ham}(3,2)$ ).
- 20 pts.    **4** Use the technique described by theorem 8.4 to find a linear code of length  $n = 6$  over  $Z_3$  with  $d = 3$  that has the most number of codewords that you can (you should be able to find a code with 27 codewords). Make sure you explain what you are doing.
- 20 pts.    **5** Use the technique described by theorem 8.4 to find a linear binary code of length  $n = 8$  with  $d = 5$  that has the most number of codewords that you can (you should be able to find a code with 4 codewords). Make sure you explain what you are doing.
- 30 pts.,  
★    **6** Consider the code generated by the parity check matrix for  $\text{Ham}(4,2)$  (this is the orthogonal code of the binary Hamming code of length 15): what is the minimum weight of this code? Suppose that we take all of the nonzero codewords of this orthogonal code and use them as the rows of an incidence matrix: show that this forms a block design. What are  $(v, k, \lambda, r, b)$  for this design? This is worth 50 pts if you can state the generalization to the orthogonal code of  $\text{Ham}(r,2)$  for any  $r$ , and you can give reasons why you think that will work.