

Math 350  
Spring, 2003

### HOMEWORK #3

Do 50 points of the following problems (due 1/30/03).

25 pts. **1** Find a necessary condition on the length  $n$  so that the binary  $(n, M, 3)$  code is perfect. What are the conditions for a perfect  $q$ -ary  $(n, M, 3)$  code?

25 pts. **2** Let  $a, b \in Z_p$  for  $p$  a prime: show that  $(a + b)^p \equiv a^p + b^p \pmod{p}$ . Explain how that can be extended to  $(a + b + \cdots + z)^p \equiv a^p + b^p + \cdots + z^p \pmod{p}$ . Use this to show that  $x^p \equiv x \pmod{p}$  for every  $x \in Z_p$ .

25 pts., **3** Consider the following matrix:  $H = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$ .  
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Show that the set of vectors  $u = (u_1, u_2, \dots, u_7)$  that satisfy  $Hu^T = (000)$  form a binary linear code. How many elements are there in this code? Use properties of the matrix  $H$  to determine the minimum distance of the code (don't just use brute force).