Math 350
Spring, 2003

**HOMEWORK #4**

Do 50 points of the following problems (due 2/6/03).

15 pts. **1** Find the n,M, and d of the binary code whose generator matrix is

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

15 pts. **2** Put the following generator matrix into standard form.

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

20 pts. **3** Let $C$ be a code. Define $C^{\perp} = \{x \in F_2^n | \sum_{i=1}^{n} x_i c_i = 0(\text{mod}2)$ for every $c \in C\}$. Find $E_n^{\perp}$, and argue why you think that you have the correct answer.

$\star$ 30 pts. **4** From the vector space $V(2,q)$, $q$ a prime power, an incidence structure $A_q$ is defined as follows: the 'points' of $A_q$ are the vectors of $V(2,q)$. The 'lines' of $A_q$ are the one-dimensional subspaces of $V(2,q)$ and their cosets. The point $P$ 'belongs to' the line $L$ if and only if $P$ is in $L$. Prove that $A_q$ is a $(v,k,\lambda,r,b) = (q^2, q, 1, q+1, q^2+q)$ design. List the points and lines of $A_2$ and $A_3$. Form the incidence matrix for $A_2$ and $A_3$ and compute the dimension and minimum distance for the binary linear codes defined by the rowspaces of these matrices.