Math 350
Spring, 2000

HOMEWORK #5

Do 50 points of the following problems (due 2/17/00).

20 pts. **1** List the (Slepian) array for the binary code with the generator matrix listed below, and decode 10101, 11111, and 00010 using your scheme. Also calculate $P_{corr}(C)$ if the symbol error probability is $p = .01$.

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

A Slepian array is the following:

00000	10011	01010	11001	00101	10110	01111	11100
00001	10010	01011	11000	00100	10111	01110	11101
00010	10001	01000	11011	00111	10100	01101	11110
10000	00011	11010	01001	10101	00110	11111	01100

With my choice of array, 10101 decodes to 00101; 11111 decodes to 01111; and 00010 decodes to 00000 (there are other natural choices). $P_{corr}(C) = (.99)^5 + 3(.99)^4(.01) = .9798079$.

15 pts. **2** Find a parity check matrix for a code that is equivalent to the linear code over Z_5 with the following generator matrix.

$$\begin{pmatrix} 1 & 0 & 2 & 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 3 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 4 & 4 \\ 0 & 2 & 2 & 0 & 4 & 0 & 1 \end{pmatrix}$$

The code coming from this generator matrix is the same as the code coming from

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 4 & 3 & 4 \\ 0 & 1 & 0 & 0 & 0 & 4 & 3 \\ 0 & 0 & 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 4 & 4 \end{pmatrix}$$

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The parity check matrix for this is

$$\begin{pmatrix} 1 & 0 & 3 & 0 & 1 & 0 & 0 \\ 2 & 1 & 4 & 1 & 0 & 1 & 0 \\ 1 & 2 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

15 pts. **3** Explain how you would decode the 3 vectors in problem 1 by using syndrome decoding.

The parity check matrix will be

H =

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

The syndromes are $10101H^T = 11; 11111H^T = 11$; and $00010H^T = 10$. The first two syndromes indicate that the received words are in the last coset, so we will add 10000 to those to decode as 00101 and 01111 respectively. The third received word is in the third coset (it is in fact the coset representative the way I have made my choices), so this will decode to 00000.

$\star$ 35 pts. **4** **a.** Show that every binary linear code has the property that either all of the codewords are even or that exactly half of them are even.

First observe that the even weight codewords will be a linear subcode of the code (simply because it will be closed under addition). Second, show that every odd weight codeword (if there are any) is in the same coset of the even weight subcode (if we add them together, we get an odd weight plus an odd weight - twice the weight of the overlap, but this is even). If all of the even weight words are in one coset and all of the odd weight cosets are in the other coset, then there must be an equal amount of these codewords, proving the result.

b. Show that in a binary linear code with the property that $C \subset C^{\perp}$, either all of the codewords have weight divisible by 4, or half of the codewords have weight that is even and not divisible by 4 and the other half have weight that is divisible by 4.

First observe that any two codewords have to overlap in an even number of positions because of the condition on $C^{\perp}$. That means that the codewords with weight divisible by 4 are a subcode of this code: they are closed under addition because the weight of the sum of any two is 4a + 4b + 2(2c). The words of even weight that are not divisible by 4 are all in the same coset, so the same argument as the previous problem applies here.