Quiz 5

Davis
M211

Name:
Pledge:

(10pts.) 1. Find $\frac{dy}{dx}$ of the following functions

a. $y = \frac{x^2+x+1}{x^3-5x^2+2}$: $\frac{dy}{dx} = \frac{(x^3-5x^2+2)(2x+1)-(x^2+x+1)(3x^2-5x)}{(x^3-5x^2+2)^2}$

b. $y = e^{x^2}\sin(x^3)$: $\frac{dy}{dx} = 2xe^{x^2}\sin(x^3) + e^{x^2}\cos(x^3)(3x^2)$

c. $y = \sin(\cos(x^2))$: $\frac{dy}{dx} = \cos(\cos(x^2))(-\sin(x^2))(2x)$

d. $y = 5\arctan(e^x)$: $\frac{dy}{dx} = \frac{5e^x}{1+(e^x)^2}$

e. $x^2y + xy^2 = 10$: $\frac{dy}{dx} = \frac{-2xy-y^2}{x^2+2xy}$

(10pts.) 2. Justify the following formulas using the given information:

a. Use $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$ together with $\lim_{h\to 0}\frac{\sin(h)}{h} = 1$ and $\lim_{h\to 0}\frac{\cos(h)-1}{h} = 0$ to show that $(\cos(x))' = -\sin(x)$.

$\lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h} = \lim_{h\to 0}\frac{\cos(x)\cos(h)-\sin(x)\sin(h)-\cos(x)}{h} = \lim_{h\to 0}\frac{\cos(x)(\cos(h)-1)}{h} - \sin(x)\frac{\sin(h)}{h} = -\sin(x)$

b. Use the chain rule to show that $(\arctan(x))' = \frac{1}{1+x^2}$.

$\tan(y) = x$; $\frac{1}{\cos^2(y)}\frac{dy}{dx} = 1$; $\frac{dy}{dx} = \cos^2(\tan^{-1}(x)) = \frac{1}{1+x^2}$ (The last part required an argument from the triangle)