Quiz 6

Davis	Name:
M211	Pledge:

(10pts.) 1. Consider the function $f(x) = xe^{-ax}$, where a is constant and $a > 0$.

a. Find any local or global maxima or minima of f (show work).

$f'(x) = x(e^{-ax}(-a)) + e^{-ax}(1) = e^{-ax}(-ax+1)$. When $f'(x)$ is set equal to 0, $x = \frac{1}{a}$ is the only critical point, and it is a global max.

b. What effect does increasing the value of a have on the positions of the maxima/minima?

As a increases, the max moves closer to the y-axis.

c. Find any inflection points of f.

$f''(x) = e^{-ax}(-a) + e^{-ax}(-a)(-ax+1) = e^{-ax}(a^2x - 2a)$. When this is set equal to 0, $x = \frac{2}{a}$ is the only inflection point.

d. On the same axes, sketch and label the graphs of f for three values of a (only sketch the curve for $x \geq 0$).

(10pts.) 2. A certain firm expects $R(x) = \ln(x)$ in revenue for x dollars spent in advertising. Suppose you want to maximize your revenue per dollar invested, or $R(x)/x$.

a. Find the marginal revenue at $x = e$

$R'(x) = \frac{1}{x}$; $R'(e) = \frac{1}{e}$.

b. Maximize the revenue per dollar invested. We need to maximize $\frac{R(x)}{x}$, so take its derivative: $\frac{xR'(x) - R(x)(1)}{x^2} = \frac{x\frac{1}{x} - \ln(x)}{x^2}$. When this is set equal to 0, we see that $x = e$ is the only critical point, and it will be a max.

c. Show on a graph of $R(x)$ why the maximum revenue per dollar invested is the same as the marginal revenue at $x = e$ (and explain in one sentence).

Show that the tangent line to $y = \ln(x)$ at $x = e$ goes through the origin. The slope of the line going from the origin through the point on the curve is the return per dollar invested, and it is a maximum when this line is tangent to the curve.