TEST 1

Davis
M211

Name:
Pledge:

Show all work; unjustified answers may receive less than full credit.

(30pts.)

1. a. You are a homicide detective, and you are told that the rate, R, at which a body cools is proportional to the temperature difference between the body and the surrounding air. If the body temperature is T, and the surrounding air is 68 degrees Fahrenheit, find a formula that gives R as a function of T. Sketch a graph of R against T.

b. The temperature at time t follows the formula $T = 68 + 30.6e^{-kt}$, where t is measured in hours after death. If the temperature is 90.6 degrees after one hour, use that information to compute k. If you find the body at 2 A.M. with a body temperature of 80, is it possible for your chief suspect to have committed the crime if she has an alibi through 10 P.M. the previous night? Explain your answer!

a. $R = k(T - 68)$

b. $90.6 = 68 + 30.6e^{-k(1)} : k = .30305.$
$80 = 68 + 30.6e^{-(.30305)t}; t = 3.0889$. The victim was killed at about 11 P.M. the previous night, and the suspect only has an alibi until 10 P.M., so she could have done it!

(30pts.)

2. a. The Banque Nationale du Zaire pays 100% annual interest on deposits, compounded monthly. You invest one million zaire (the currency in Zaire): how much money do you have after 1 year?

b. Repeat the computation for compounding daily and hourly (still with 100% annual interest): how much money would you have after a year in these cases?

c. Would the amount increase without bound if you continued to compound more and more often? Either give a reason or give an upper bound on the amount of money you would have at the end of a year.

d. If you compound continuously, how long does it take to double your money?

a. $(1 + 1/12)^{12} = 2.613$ million zaires.

b. $(1 + 1/365)^{365} = 2.715$ million zaires; $(1 + 1/8760)^{8760} = 2.718$ million zaires.

c. The upper bound for the amount of zaires is e million.

d. $2 = e^t : t = ln2$ years, or a little more than 8 months.

(25pts.)

3. The amount of daylight in Richmond varies from a low of 10 hours on December 21 to a high of 14 hours on June 21. Find a mathematical equation that would tell you the number of hours of daylight at any time t, and use your equation to estimate the number of hours of sunlight on September 25. The amount of daylight in Bristol, England varies from a low of 8 hours on December 21 to a high of 16 hours on June 21: find an equation to describe this. Graph both functions on the same axes, and label the point of intersection. If Brazil is roughly the same distance south of the equator as Richmond is North, what equation will describe the number of hours of daylight in Brazil?

$D = 12 - 2\cos\frac{2\pi}{365}(t-10)$ for Richmond, $D = 12 - 4\cos\frac{2\pi}{365}(t-10)$ for England, and $D = 12 + 2\cos\frac{2\pi}{365}(t-10)$ for Brazil.

(15pts.)

4. A weight hanging off the end of a spring will bob up and down, eventually coming to a stop. If its motion can be described by the equation $h = (.9)^t \sin t$, where t is measured in seconds and h is measured in feet, approximately how fast is the weight moving at time $t = 1$? Draw the graph of the equation and show how you are getting your approximation on the graph, and show all your work!

t	h
.9	.71246
.99	.7532
.999	.7569
1.001	.75773
1.01	.761346
1.1	.79368

The slope between the closest points in this chart is $\frac{.75773-.7569}{1.001-.999} = .415$. The "actual" value for the slope of this tangent line is .406, but we won't be able to get that value until later this semester.