TEST 1

Davis
M211

Name:
Pledge:

Show all work; unjustified answers may receive less than full credit.

(20pts.)

1. a. When a sports car slams on its brakes, the velocity v (in mph) is in a linear relationship with the time t (in seconds). If it takes 7 seconds for a car going 70 miles an hour to come to a complete stop, find an equation for velocity in terms of time (in seconds after the brake is applied). Sketch a graph of velocity against time. How fast is the car going at $t = 2$?

b. According to the April 1991 issue of *Car and Driver*, an Alfa Romeo going at 70 mph requires 177 feet to stop. Assuming that the stopping distance is proportional to the square of the velocity, find the stopping distance for an Alpha Romeo going 140 mph (its top speed).

a. $v = 70 - 10t$; at $t = 2$ seconds, the car is going 50 mph.

b. $d = kv^2; 177 = k(70)^2; k = 177/(70)^2 : d = (177/(70)^2)(140)^2 = 4(177) = 708$. An Alpha Romeo going 140 mph requires 708 feet to stop.

(20pts.)

2. In an article in the New York Times, descendents of a wealthy Revolutionary War merchant claim that they should be repaid for a loan made to the Continental Congress. Jacob DeHaven lent $450,000 to the Congress in 1777. If we assume that the loan should be repaid at a 6% annual rate, how much does the US owe the family? How long does it take for the debt to double?

$P = 450000(1.06)^t; 1998 - 1777 = 221$ years; $P = 450000(1.06)^{221} = 1.76 \times 10^{11}$. THe US owes the family about 176 billion dollars. To figure out how long it takes to double the debt, use $2P_0 = P_0(1.06)^t; t = ln2/ln(1.06) = 11.9$ years. The debt doubles about every 12 years.

(20pts.)

3. From the table below, determine a possible formula for each function.

x	f(x)	g(x)	h(x)	F(x)
-5	.09375	.9589	14	-125
-4	.1875	.7568	12.5	-64
-3	.375	-.1411	11	-27
-2	.75	-.9093	9.5	-8
-1	1.5	-.8415	8	-1
0	3	0	6.5	0
1	6	.8415	5	1
2	12	.9093	3.5	8
3	24	.1411	2	27
4	48	-.7568	.5	64
5	96	-.9589	-1	125

$f(x) = 3 \cdot 2^x; g(x) = \sin x; h(x) = 6.5 - 1.5x; F(x) = x^3$

(20pts.)

4. Given the graph of $y = f(x)$ below, sketch graphs of the following (make sure you show the scale on your graphs):

a. $y = 5 + f(x)$

b. $y = f(x - 3)$

c. $y = -2f(x)$

d. $y = f(.5x)$

a. Shifts the graph up 5 units.

b. Shifts right 3 units.

c. Stretches by a factor of 2 in the y-direction and then flips the picture about the x-axis.

d. Stretches by a factor of 2 along the x-axis.

(20pts.) **5.** Sketch a graph of $y = \sin x$. Find the slope of the line through the points on the curve whose x-coordinates are $\pi/6$ and $\pi/4$, and show this line on your sketch. Find the slope of the line through the points whose x-coordinates are $\pi/4 - .01$ and $\pi/4$. Do the same for $\pi/4$ and $\pi/4 + .01$, and use this to estimate the slope of the tangent line to $y = \sin x$ at $\pi/4$. Set up **BUT DO NOT EVALUATE** the limit equation to find the slope of the tangent line at $\pi/4$.

Slope $= \frac{\sin\frac{\Pi}{4} - \sin\frac{\Pi}{6}}{\frac{\Pi}{4} - \frac{\Pi}{6}} = .791$. Similar computations for the others yield slopes of about .7 and .71, so .705 is a reasonable guess for the slope at $\frac{\Pi}{4}$. The limit equation is $lim_{h\to 0}\frac{\sin(\frac{\Pi}{4}+h) - \sin\frac{\Pi}{4}}{h}$.