TEST 3

Davis Name:
M211 Pledge:

Show all work; unjustified answers may receive less than full credit.

(20pts.) **1.** Calculate the slope of the tangent line to the curve $xy^2 + x^2y = 2$ at the point $(1, 1)$ (hint: use implicit differentiation). Write the equation of the tangent line at that point, and use the tangent line approximation to the curve at $(1, 1)$ to estimate the y value of the curve at $x = 1.05$.

(20pts.) **2.** Consider the family of functions of the form $y = x^4 + ax^2$. Explain why this function has three distinct critical points when $a < 0$. Sketch the graph of this function when $a = -8$, showing all relevant points.

(40pts.) **3.** Suppose the marginal profit satisfies the equation $\pi'(q) = -q^3/3 + 250q^2$.

a. What positive value of q will maximize the profit?

b. What positive value of q will maximize the marginal profit?

c. Estimate the area under the marginal profit curve from $q = 0$ to $q = 100$ using 4 subintervals for both a left hand sum and a right hand sum. What does this number represent?

d. If $\pi(q) = -q^4/12 + (250/3)q^3$, use the Fundamental Theorem of Calculus to get the exact area under the marginal profit curve from $q = 0$ to $q = 100$. (hint: think of π as F)

(20pts.) **4.** Find the volume of the largest box that can be made from a piece of cardboard, 8 inches square, by cutting equal squares from each of the corners and turning up the sides.

Have a great Thanksgiving: you have earned it!