TEST 3

Davis
M211

Name:
Pledge:

Show all work; unjustified answers may receive less than full credit.

(20pts.) **1.** Compute $\frac{dy}{dx}$ of the following functions:

a. $y = \frac{e^x \sin(x)}{\ln(x)}$ $\frac{dy}{dx} = \frac{\ln(x)(e^x \cos(x)+e^x \sin(x))-\frac{1}{x}(e^x \sin(x))}{(\ln(x))^2}$

b. $y = x^2 + 2^x + 5$ $\frac{dy}{dx} = 2x + \ln(2)2^x$

c. $y = \sin(\arcsin(e^x))$ $\frac{dy}{dx} = e^x$

d. $y = \cos(\frac{x^3+2}{x-5})$ $\frac{dy}{dx} = -\sin(\frac{x^3+2}{x-5})(\frac{(x-5)(3x^2)-(x^3+2)(1)}{(x-5)^2})$

e. $x\ln(y) + y\tan(x) = 2$ $\frac{dy}{dx} = \frac{-\ln(y)-\frac{y}{\cos^2(x)}}{\frac{x}{y}+\tan(x)}$

(20pts.) **2.** Argue that $(f(x)g(x))' = f(x)g'(x) + g(x)f'(x)$. (Hint: You will get 12 pts. if you are able to sketch the appropriate picture and label it correctly).

$$\lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} = \lim_{h\to 0} f(x+h)\frac{g(x+h)-g(x)}{h} + \lim_{h\to 0} g(x)\frac{f(x+h)-f(x)}{h}$$

This implies that $(f(x)g(x))' = f(x)g'(x) + g(x)f'(x)$.

(20pts.) **3.** Consider the function $f(x) = x^3 + ax$, where a is a constant.

a. What values of a will yield two distinct critical points? When $a < 0$, then $f'(x) = 3x^2 + a$ will have two distinct roots, and hence two distinct critical points.

b. Are there any inflection points of this function? $x = 0$ is an inflection point since the second derivative is 0 at $x = 0$, it is negative for $x < 0$, and it is positive for $x > 0$.

c. On the same axes, sketch $f(x)$ for three different values of a, at least one of which produces two distinct critical points. Make sure you carefully label your graph.

(20pts.) **4.** A rectangular page is to contain 24 square inches of print. The top and bottom margins are 1.5 inches, while the side margins are 1 inch. What should the dimensions of the page be so that the least amount of paper is used?

Call x the dimension of the *print* across the bottom, and y the dimension of the *print* up the side. The condition of the problem implies that $xy = 24$. We want to minimize $A = (x+2)(y+3) = 24 + 3x + \frac{48}{x} + 6$.

We need to compute $\frac{dA}{dx} = 3 - \frac{48}{x^2}$ and set this to 0. The solution to this equation is $x = 4$, and that implies $y = 6$. Thus, the dimensions of the paper are 6 by 9.

(20pts.) **5.** Suppose the marginal cost of producing calculus textbooks is $C'(q) = -.001q^2 + 30$, where q is measured in thousands of texts and $C(q)$ is measured in thousands of dollars. Use the Fundamental Theorem of Calculus to find the total cost of making $q = 150$ textbooks (starting at $q = 0$). If the marginal revenue is $R'(q) = 26$, how much revenue do you make from $q = 0$ to $q = 150$? Sketch a graph with the marginal cost and the marginal revenue from $q = 0$ to $q = 150$, and show on your picture the value of q where you start to make a profit (you may do this algebraically as well).

The cost of producing $q = 150$ textbooks is $\int_0^{150} C'(q)dq = \frac{-.001q^3}{3} + 30q|_0^{150} = 3375$. The revenue produced is $\int_0^{150} R'(q)dq = 26q|_0^{150} = 3900$. A profit is made when the revenue produced is more than the cost, and this happens at about $q = 109$.

Have a great Thanksgiving: you have earned it!