**Test # 1**

Take home test (closed book, 2:30 hours)

Davis
M245

Name:
Pledge:

10pts **1.** Give an example of a system of three equations with three variables so that:

**A.** There is a unique solution (how do you know the solution is unique?).

**B.** There are infinitely many solutions (how do you know there are infinitely many solutions?).

**C.** There are no solutions (how do you know there are no solutions?).

10pts **2.** Consider the matrix $\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 5 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix}$. Row reduce the matrix to reduced echelon form. Is the vector $(2,1,1,0)$ in the span of the columns of the matrix? Explain your answer.

10pts **3.** Describe the solution set for the homogeneous system of equations

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 0 \\ 2x_1 + 4x_2 + 5x_3 - 2x_4 &= 0 \end{aligned}$$

Compare it with the solution set for the system

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 4 \\ 2x_1 + 4x_2 + 5x_3 - 2x_4 &= 7 \end{aligned}$$

10pts **4.** Suppose $w_1$ and $w_2$ are in $Span\{u,v\}$: argue that $cw_1 + dw_2$ is in $Span\{u,v\}$ for any weights $c$ and $d$.

10pts **5.** Use Mathematica to decide if the linear transformation $T$ defined by $T(\mathbf{x}) = A\mathbf{x}$, where $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 15 & -2 & 0 & 13 \\ -5 & 12 & 19 & -25 \\ 22 & 33 & 44 & 55 \\ 7 & 8 & 9 & 10 \end{pmatrix}$, is a 1-1 transformation (make sure you include the definition of 1-1 in your answer to this question).

10pts **6.** Sketch the image of the house after it is transformed by the following linear transformations (start with the picture of the house in the first quadrant as we did in class):

**A.** $T(\mathbf{x}) = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \mathbf{x}$

**B.** $T(\mathbf{x}) = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x}$

**C.** $T(\mathbf{x}) = \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix} \mathbf{x}$

10pts **7.** Let $T$ be a linear transformation and let $\{v_1, v_2, v_3\}$ be a linearly dependent set in $R^n$. Show that the set $\{T(v_1), T(v_2), T(v_3)\}$ is linearly dependent.

10pts **8.** Find the inverse of the matrix $\begin{pmatrix} 1 & 0 & 3 \\ -1 & 1 & -3 \\ 2 & 0 & 7 \end{pmatrix}$ and use that inverse to get a solution to the system of equations

$$\begin{array}{rcrcrcr} x_1 & & & + & 3x_3 & = & 2 \\ -x_1 & + & x_2 & - & 3x_3 & = & 3 \\ 2x_1 & & & + & 7x_3 & = & 4. \end{array}$$

Is the solution unique? Explain.

20pts **9.** Mark each of the following statements either True or False. Justify each answer.

**A.** An augmented matrix with a pivot in every row is associated to a consistent system of linear equations.

**B.** A linear transformation is completely determined by its effect on the vectors $e_1, e_2, \ldots, e_n$.

**C.** Each column of $AB$ is a linear combination of the columns of $A$ using weights from the corresponding column of $B$.

**D.** A product of invertible $n \times n$ matrices is invertible, and the inverse of the product is the product of their inverses.

**E.** If the columns of an $n \times n$ matrix are linearly dependent, then the columns span $R^n$.

5pts **BONUS PROBLEM:** Suppose $w_1$ and $w_2$ are *not* in $Span\{u, v\}$: $w_1 + w_2$ is *not* in $Span\{u, v\}$.