Test # 2

Take home test (closed book, 2:30 hours)

Davis
M245

Name:
Pledge:

10 pts. **I.** Which of the following matrices have the same column space? Which have the same null space? List all pairings, and briefly justify your answers.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}, C = \begin{pmatrix} 1 & 2 \\ 5 & 6 \end{pmatrix}, D = \begin{pmatrix} 1 & 2 \\ 3 & 6 \\ 5 & 10 \\ 7 & 9 \end{pmatrix},$$

$$E = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix}, F = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 5 \end{pmatrix}$$

10pts. **II.** Find a basis for $NulA, ColA$, and $RowA$ of the following matrix. State the rank theorem and verify it for this matrix.

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 6 & 9 & 11 & 13 \\ 3 & 6 & 9 & 12 & 15 & 20 \\ 4 & 8 & 12 & 21 & 25 & 35 \end{pmatrix}$$

10pts. **III.** Let V be a vector space and let $\{v_1, v_2, \ldots, v_p\}$ be a collection of vectors in V. Show that $Span\{v_1, v_2, \ldots, v_p\}$ is a subspace of V.

10pts. **IV.** Show that the following collection of vectors is a subspace or give a counterexample why it is not a subspace (be careful!).

$$W = \{ \begin{bmatrix} a + 4b + 7 \\ 2a + 5b + 8 \\ 3a + 6b + 9 \end{bmatrix} : a, b \text{ real}\}$$

15pts. **V.** Compute the determinant of the matrices $E(n)$, where $E(n)$ is the matrix with 2's down the main diagonal and -1's on the diagonals to either side of the main diagonal, for $n = 2, 3, 4, 5$ and 6. For example, $E(5) = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{pmatrix}$. What is the determinant of $E(100)$? Full credit only if you can justify your answer (explain what you have done).

10pts. **VI.** Use the concept of volume to explain why the determinant of a 3×3 matrix A is zero if and only if A is not invertible. Do not just quote the theorem from the book which states that this is true: I want a geometrical explanation why it is true. [Hint: Think about the columns of A.]

10pts. **VII.** Let $\mathcal{B} = \{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}\}$ and $\mathcal{C} = \{\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix}\}$ be bases for R^2. Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$ and from $\mathcal{C}$ to $\mathcal{B}$. If $\mathbf{x} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$, find $[\mathbf{x}]_{\mathcal{B}}$ and $[\mathbf{x}]_{\mathcal{C}}$.

10pts. **VIII.** Use Mathematica to find a basis for the space spanned by the following vectors. Is this space equal to R^7? Explain your answer. $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 8 \\ 2 \\ 15 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 6 \\ -10 \\ 4 \\ -20 \\ 12 \\ 14 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 4 \\ 2 \\ 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 31 \\ 16 \\ 20 \\ 11 \\ 22 \end{bmatrix}.$

20pts. **IX.** Answer the following true/false questions. In each case, either give a brief (two complete sentences) justification for your answer or provide a counterexample.

A. The columns of an invertible $n \times n$ matrix form a basis for R^n.

B. If A is a 2×2 matrix with determinant 0, then the columns are the scalar multiples, or a column is zero.

C. $Col A$ is the set of all vectors that can be written as Ax for some x (A is a $m \times n$ matrix).

D. Let $\mathcal{B}$ be a basis for a vector space R^n. If $P_{\mathcal{B}}$ is the change-of-coordinates matrix, then $\mathbf{x} = P_{\mathcal{B}}[\mathbf{x}]_{\mathcal{B}}$ for any $\mathbf{x} \in R^n$.

E. If $\{v_1, \ldots, v_p\}$ are a set of linearly independent vectors from a vector space V and T is a linear transformation from V to W, then $\{T(v_1), \ldots, T(v_p)\}$ are linearly independent in W.