Quiz 7

Davis
M212

Name:
Pledge:

(5pts.) 1. Verify that $y = c_0\frac{e^x+e^{-x}}{2} + c_1\frac{e^x-e^{-x}}{2}$ is a solution of the differential equation $y'' - y = 0$.

We simply need to take the appropriate derivatives and plug them into the differential equation. We get:

$$y' = \frac{c_0}{2}(e^x - e^{-x}) + \frac{c_1}{2}(e^x + e^{-x}); y'' = \frac{c_0}{2}(e^x + e^{-x}) + \frac{c_1}{2}(e^x - e^{-x})$$

Thus, $y'' - y = \frac{c_0}{2}(e^x + e^{-x}) + \frac{c_1}{2}(e^x - e^{-x}) - (c_0\frac{e^x+e^{-x}}{2} + c_1\frac{e^x-e^{-x}}{2}) = 0$.

(10pts.) 2. Find the first 6 nonzero terms of the series solution to $y'' - y = 0$.

The set-up is $y = c_0 + c_1x + c_2x^2 + c_3x^3 + \cdots; y' = c_1 + 2c_2x + 3c_3x^2 + 4c_4x^3 + \cdots; y'' = 2c_2 + 6c_3x + 12c_4x^2 + 20c_5x^3 + \cdots$; the coefficient equations are $2c_2 - c_0 = 0; 6c_3 - c_1 = 0; 12c_4 - c_2 = 0; 20c_5 - c_3 = 0$. Solving this yields $c_2 = \frac{c_0}{2}; c_3 = \frac{c_1}{6}; c_4 = \frac{c_2}{12} = \frac{c_0}{24}; c_5 = \frac{c_3}{20} = \frac{c_0}{120}$. We could continue, but you are only asked to find the first 6 nonzero terms of the series solution, which will be $y = c_0 + c_1x + \frac{c_0}{2}x^2 + \frac{c_1}{6}x^3 + \frac{c_0}{24}x^4 + \frac{c_1}{120}x^5 + \cdots$.

(5pts.) 3. Use the MacLaurin series for e^x to find the first 6 nonzero terms of the power series for $c_0\frac{e^x+e^{-x}}{2} + c_1\frac{e^x-e^{-x}}{2}$.

Since $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$, we get that $e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \cdots$. Combining this we get $c_0\frac{e^x+e^{-x}}{2} + c_1\frac{e^x-e^{-x}}{2} = \frac{c_0}{2}(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots + (1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \cdots)) + \frac{c_1}{2}(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots - (1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \cdots)) = c_0(1 + \frac{x^2}{2} + \frac{x^4}{4!} + \cdots) + c_1(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots)$. Notice that this is the same as the answer to problem 2, confirming that the proposed solution in problem 1 really is the solution to the differential equation.