# Quiz 5

Davis
M212

Name:
Pledge:

(10pts.) 1. An aquarium pool has volume $2 \times 10^6$ liters. The pool initially contains pure fresh water. At $t = 0$ minutes, water containing 10 grams/liter of salt is poured into the pool at a rate of 60 liters/minute. The salt water instantly and totally mixes with the fresh water, and the excess mixture is drained out of the pool at the same rate (60 liters/minute).

**a.** Write a differential equation for the amount of salt in the pool.

$\frac{dQ}{dt} = 60(10) - \frac{Q}{2\times 10^6}(60)$

**b.** Solve the differential equation (use information in the problem regarding the initial condition).

$\int \frac{dQ}{60(10)-\frac{Q}{2\times10^6}(60)} = \int dt; -\frac{2\times10^6}{60}\ln\left(60(10) - \frac{Q}{2\times10^6}(60)\right) = t+C; Q = 2\times10^7 - Ae^{-3\times10^{-5}t}; A = 2\times10^7.$

**c.** What happens to the amount of salt as $t \to \infty$?

The quantity approaches $2 \times 10^7$, or the concentration approaches 10 grams per liter.

2.

(10pts.) **a.** Solve the following initial value problem: $y'' + 4y' + 3y = 0, y(0) = -1, y'(0) = 0$.

The characteristic equation is $r^2 + 4r + 3 = (r+3)(r+1) = 0; r = -1, -3$. The general solution is $y = C_1e^{-3t} + C_2e^{-t}$. Using the initial value, we get $-1 = C_1 + C_2$ and $0 = -3C_1 - C_2$. Adding those equations together, we get $-1 = -2C_1$, so $C_1 = \frac{1}{2}$ and $C_2 = -\frac{3}{2}$. Thus, $y = \frac{1}{2}e^{-3t} - \frac{3}{2}e^{-t}$.

**b.** Verify that your solution satisfies the differential equation.

The first derivative is $y' = -\frac{3}{2}e^{-3t} + \frac{3}{2}e^{-t}$ and the second derivative is $y'' = \frac{9}{2}e^{-3t} - \frac{3}{2}e^{-t}$. When we plug this into the differential equation, we get $\frac{9}{2}e^{-3t} - \frac{3}{2}e^{-t} + 4(-\frac{3}{2}e^{-3t} + \frac{3}{2}e^{-t}) + 3(\frac{1}{2}e^{-3t} - \frac{3}{2}e^{-t}) = (\frac{9}{2} - 4(\frac{3}{2}) + 3(\frac{1}{2}))e^{-3t} + (-\frac{3}{2} + 4(\frac{3}{2}) - 3(\frac{3}{2}))e^{-t} = 0$.

**c.** Suppose this differential equation represents the motion of a mass on a spring: explain the physical meaning of 2 of the numbers in the equation in part a. (extra credit if you can explain the physical meaning of all of the numbers).

The 3 in front of the $y$ term is the constant of proportionality from Hooke's Law; the 4 in front of the first derivative term is the constant due to air resistance; the -1 in the initial condition is the initial displacement; and the 0 in the initial condition is the initial velocity. Thus, this is a mass on a spring that is displaced 1 unit and then released.