Fourth Homework Assignment

9/17: 1,5,9
9/20: 13,15,27
9/22: 31,35,39
9/24: 45,51,59

Turn-in problems due 9/24: 4,8,12,20,28,30,38,42,50,58

⋆ **problem**: The Chinese remainder theorem states that two relatively prime ideals I_1 and I_2 of a ring R (meaning $I_1 + I_2 = R$) and two ring elements $a_1, a_2 \in R$ must have a third ring element $a \in R$ satisfying $a \equiv a_i \bmod I_i$. Use this to show that if I_1 and I_2 are relatively prime and $I = I_1 \cap I_2$, then R/I is isomorphic to $R/I_1 \bigoplus R/I_2$. If we define the Euler ϕ-function to be the number of positive integers less than k that are relatively prime to k, and m and n are relatively prime integers, then use the previous result to show that $\phi(mn) = \phi(m)\phi(n)$. Show that if p is a prime then $\phi(p^e) = p^e - p^{e-1}$. Hence, prove that if $n = p_1^{e_1} p_2^{e_2} \cdots p_r^{e_r}$ for p_i distinct primes, then $\phi(n) = \prod_{i=1}^{r}(p^{e_i} - p^{e_i-1}) = n \prod_1^r (1 - \frac{1}{p_i})$.